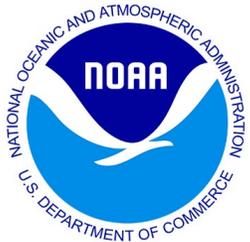


# Modeling bubble mediated gas transfer by breaking waves



**PRINCETON**  
School of Engineering and Applied Science

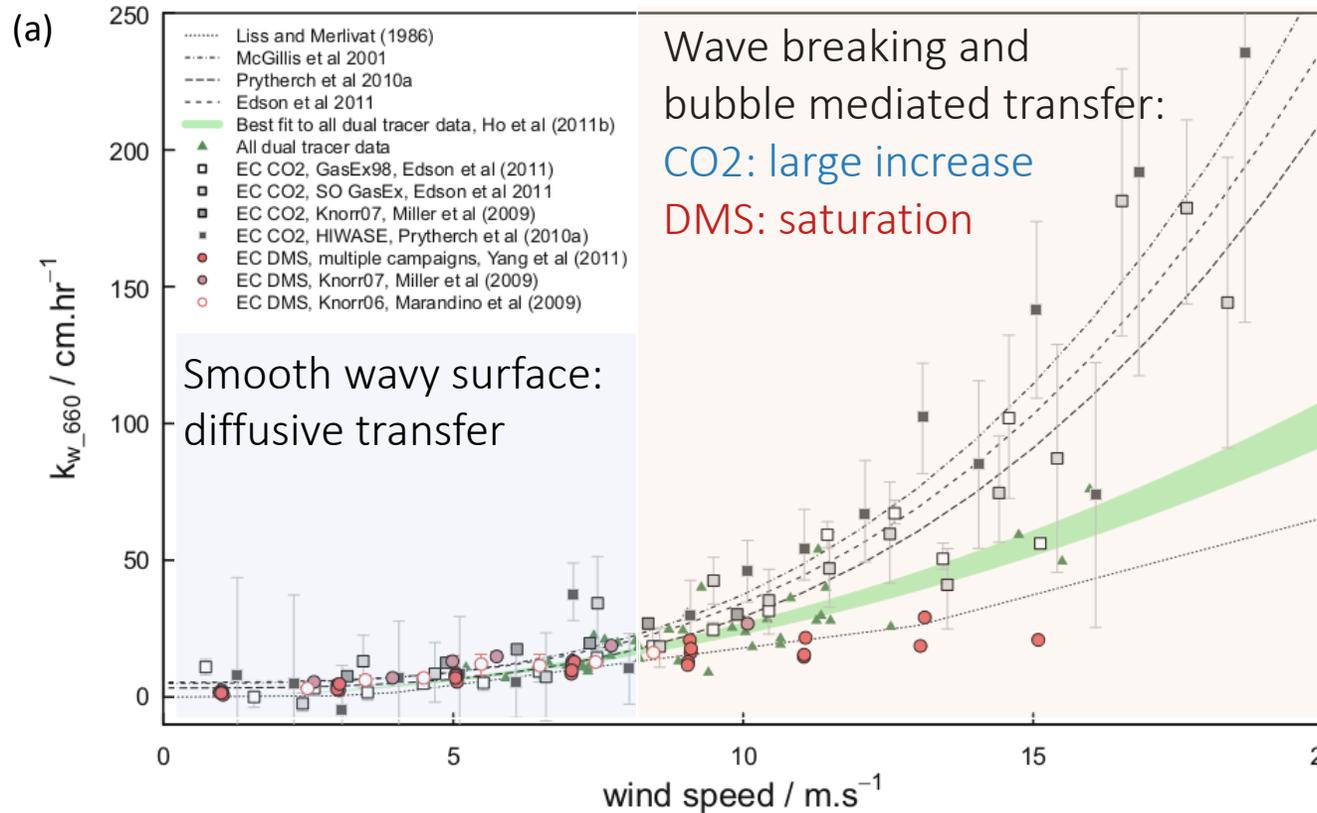


 High Meadows  
Environmental  
Institute

**Luc Deike**

Department of Mechanical and Aerospace Engineering, and  
High Meadows Environmental Institute, Princeton University  
With W. Mostert, P. Kumar Farsoiya, D. Ruth, A. Riviere, S. Perrard, J. Wu,  
B. Reichl<sup>2</sup> (<sup>2</sup> NOAA GFDL), S. Popinet<sup>3</sup> (<sup>3</sup> Sorbonne University).

# Field measurement of the gas transfer velocity



-- Various  
parameterizations

$$k_w \propto Sc^{-n} U_{10}^m$$

$$m \approx 2$$

*Garbe et al 2014*  
*Brumer et al 2017*  
*Bell et al 2017*

Wind speed is not enough to describe the transfer of gas

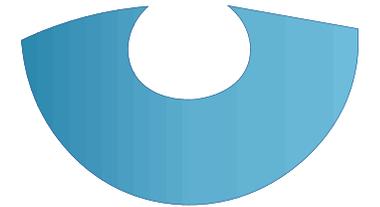
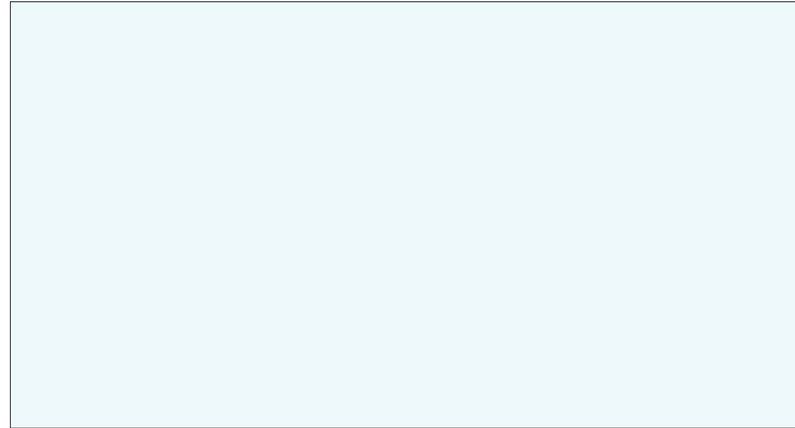
How can we incorporate the role of breaking and associated bubbles in gas transfer estimations?

# A multi-scale approach for ocean-atmosphere interaction

$O(1\text{m}-10\text{km})$

$O(1\text{mm}-10\text{m})$

$O(1\mu\text{m}-10\text{mm})$



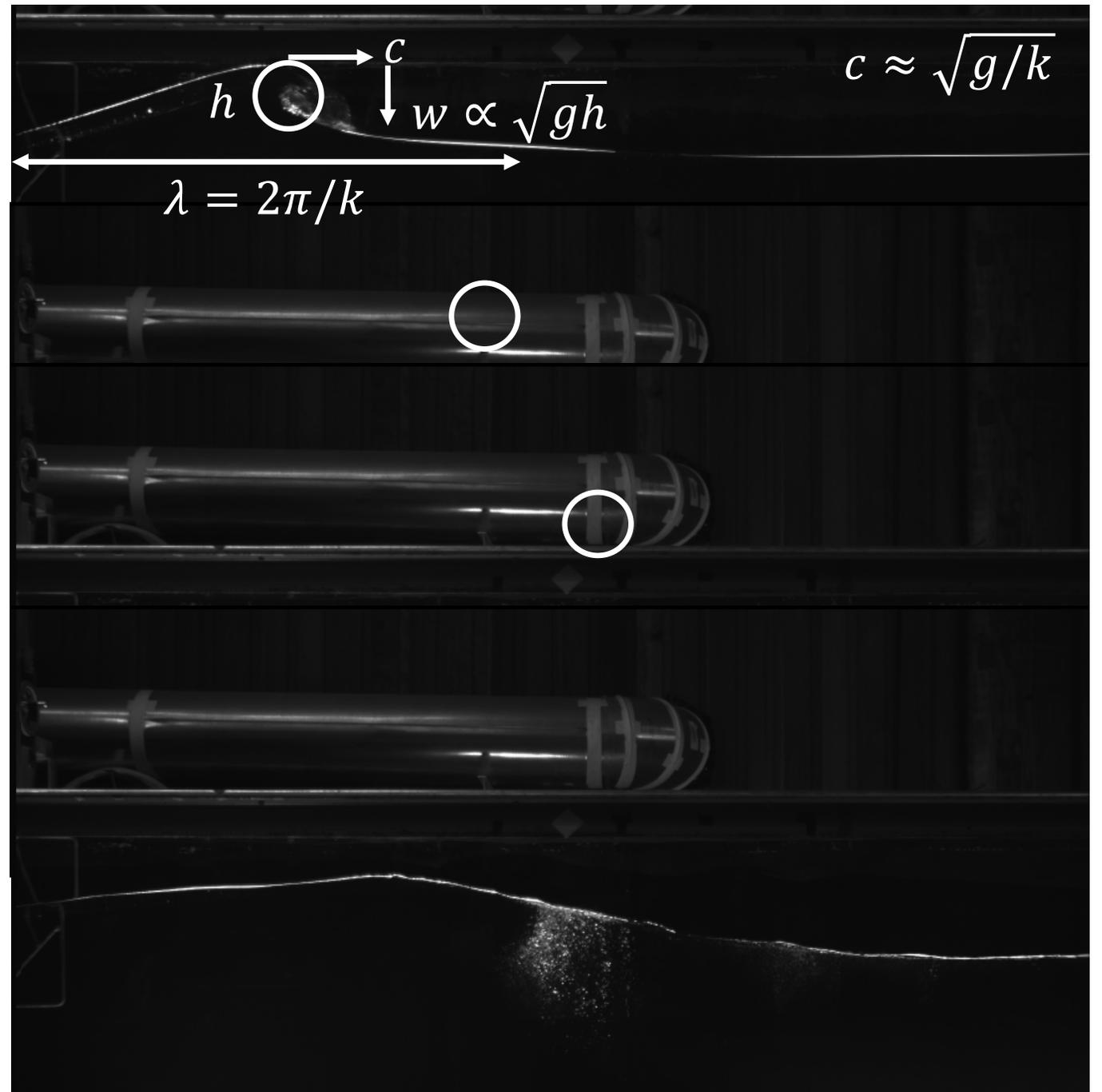
# The scales of breaking waves

Laboratory experiments by W.K. Melville;  
J. Duncan; M. Banner, M. Perlin, etc...

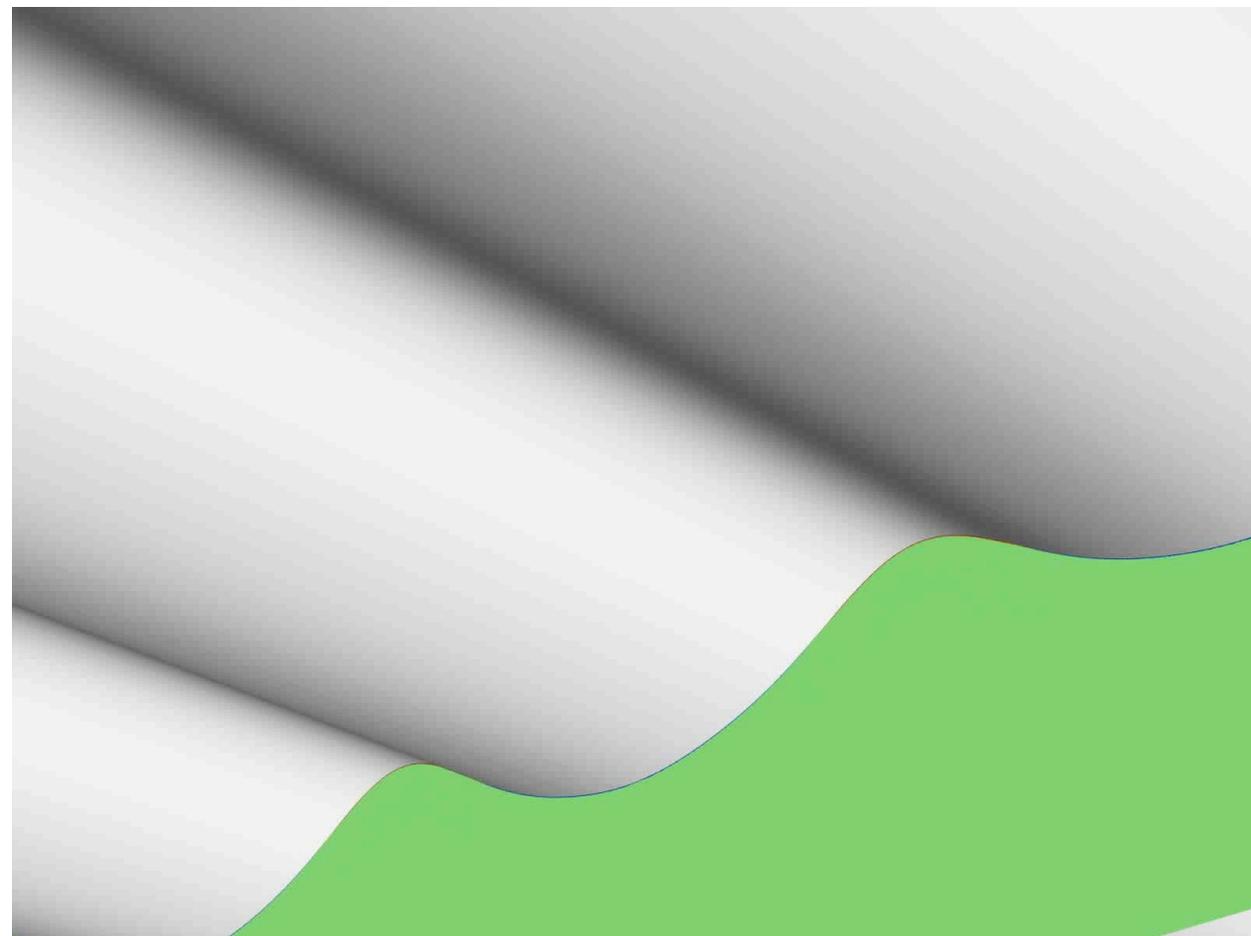
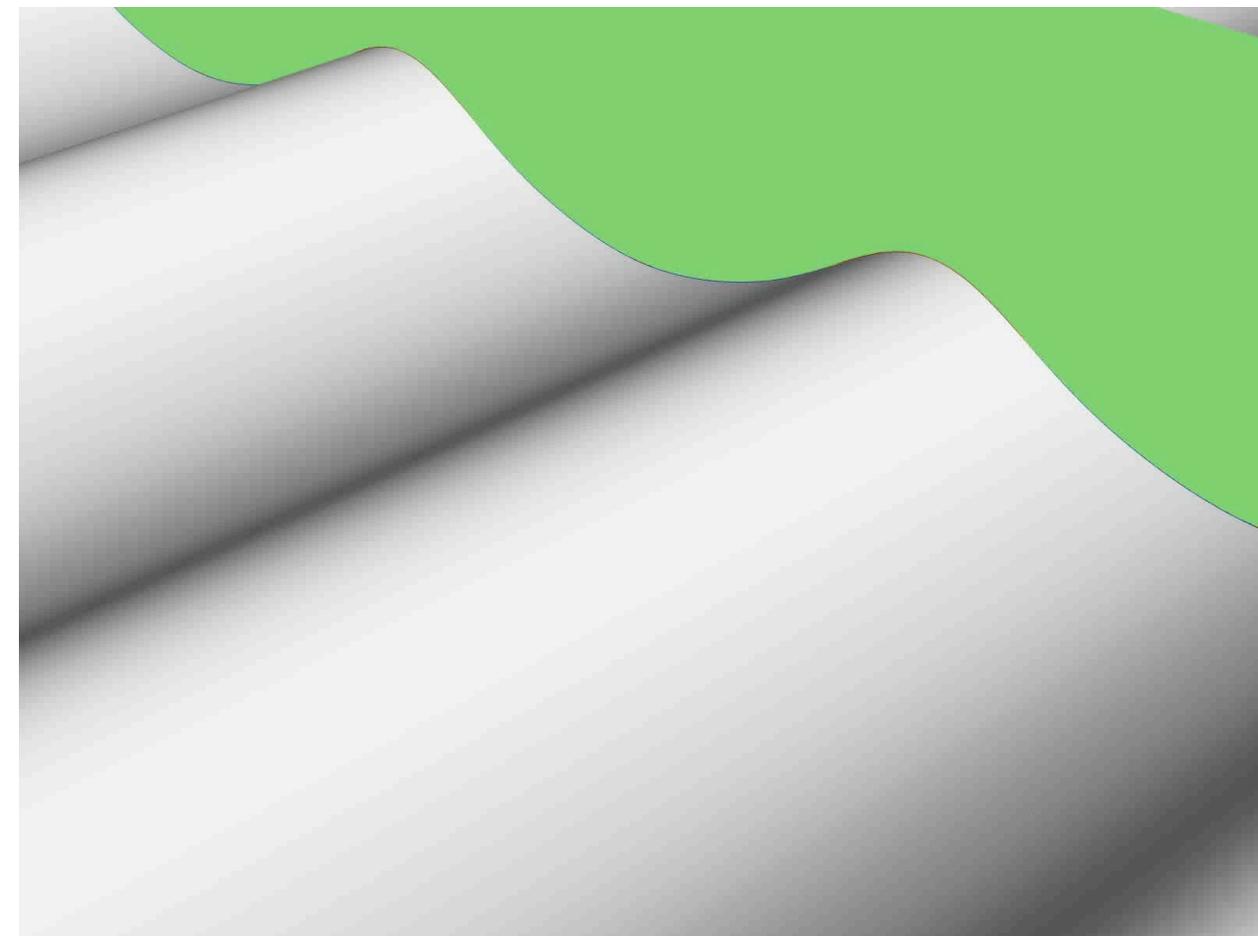
Energy dissipation rate by breaking:  
(Drazen et al 2008, Romero et al 2012,  
Deike et al 2015, 2016)

$$\varepsilon \propto \frac{w^3}{h}$$

$$\varepsilon_l \propto \frac{\rho}{g} (hk)^{5/2} c^5$$



# Direct Numerical Simulations of breaking waves (DNS)

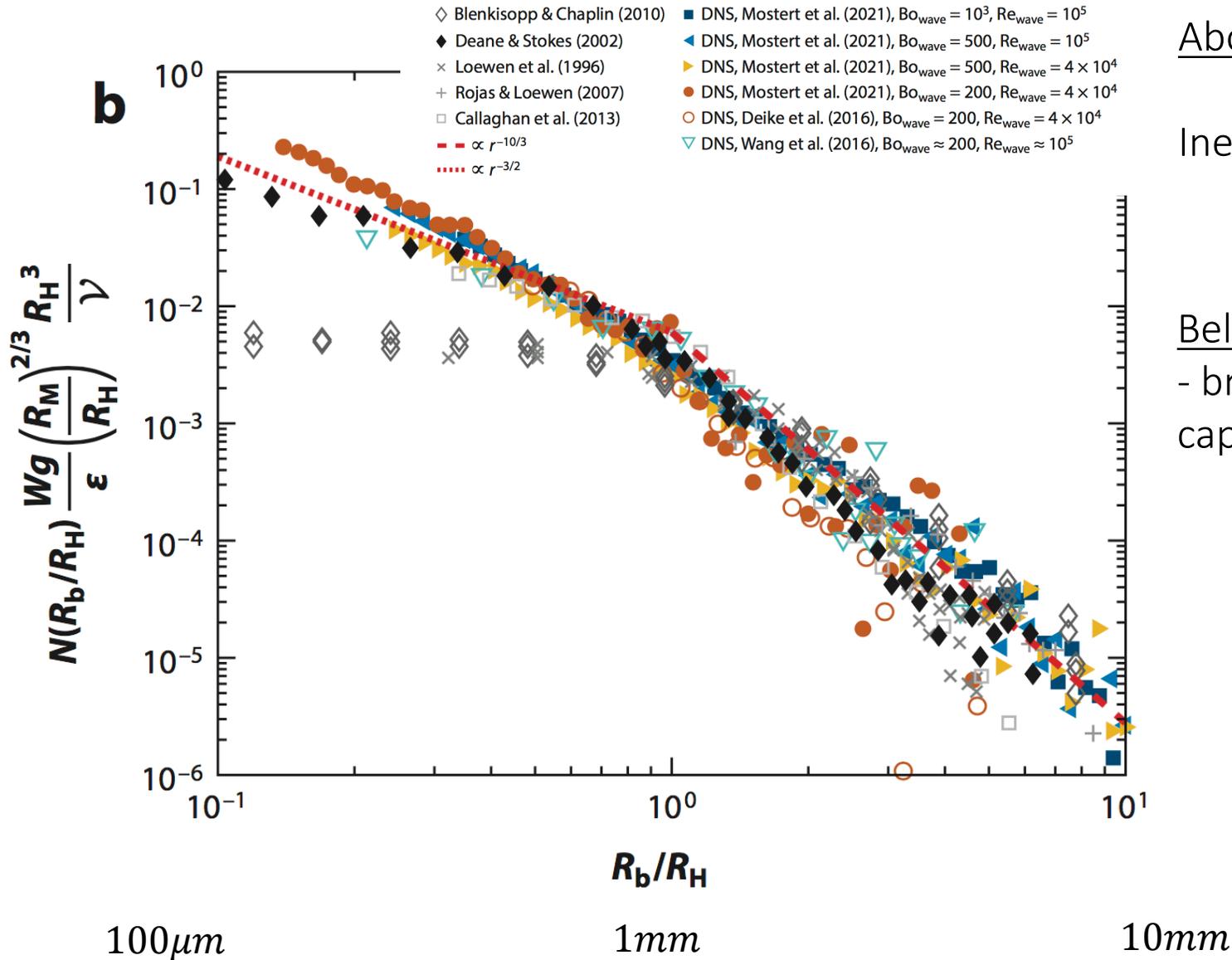


Basilisk Flow Solver Open source (S. Popinet) <http://basilisk.fr>

*Deike, Melville and Popinet 2016,  
Mostert, Popinet and Deike, 2022*

**Air entrainment, drops and bubble statistics**

# Bubble size distribution under a breaking wave



Above the Hinze scale:

Inertial turbulent break-up (*Garrett et al 2000*)

$$n(R_b) \sim R_b^{-10/3}$$

Below the Hinze scale

- break-up of large bubbles and driven by capillary break-up of elongated filaments

$$n(R_b) \sim R_b^{-3/2}$$



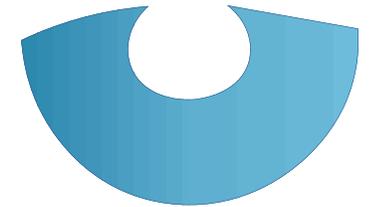
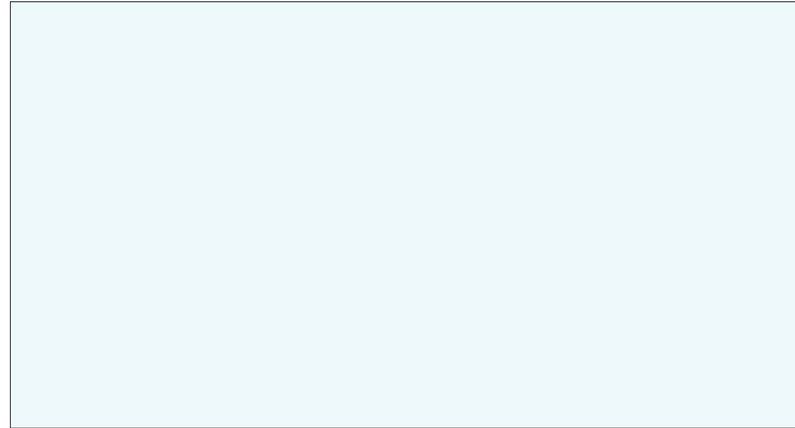
See Mostert, Popinet and Deike 2022,  
 Ruth et al, in review,  
 Riviere et al, in review  
 And Dan Ruth's talk yesterday

# A multi-scale approach for ocean-atmosphere interaction

$O(1\text{m}-10\text{km})$

$O(1\text{mm}-10\text{m})$

$O(1\mu\text{m}-10\text{mm})$



# Mass (gas) transfer of an individual bubble in a turbulent flow



Higbie' 1938 eddy renewal theory:  $k_L \propto \sqrt{\frac{D}{\theta}}$

For a bubble/interface in a turbulent flow:  $\theta \propto \frac{\eta_K}{u'_k}$

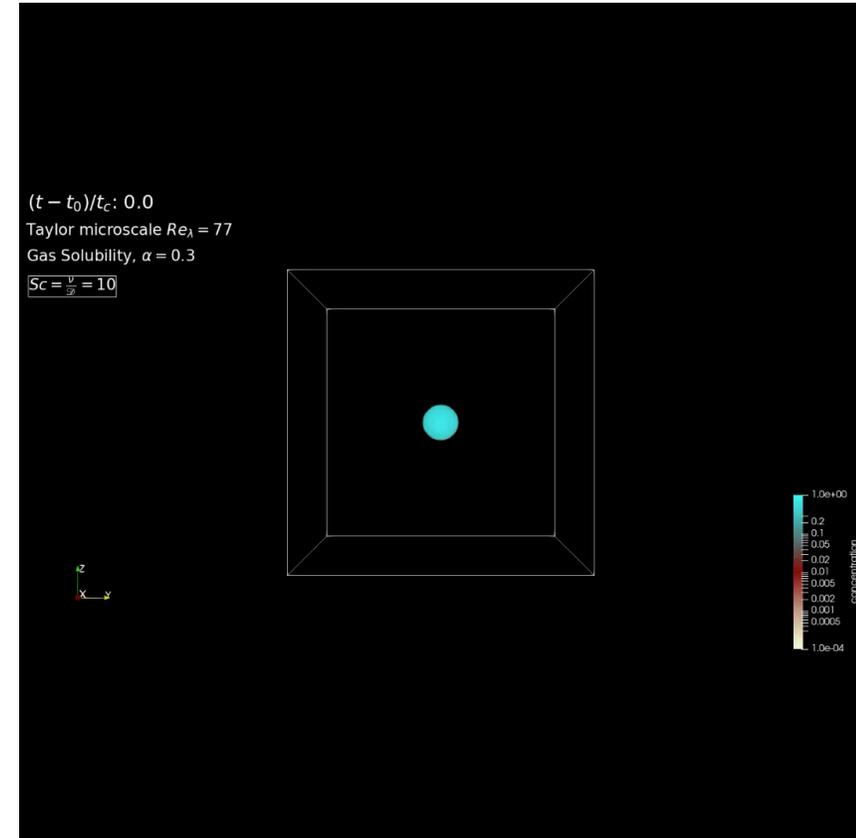
The transfer rate then reads:

$$k_L \propto Sc^{-1/2} (\varepsilon \nu)^{1/4}$$

In non dimensional form

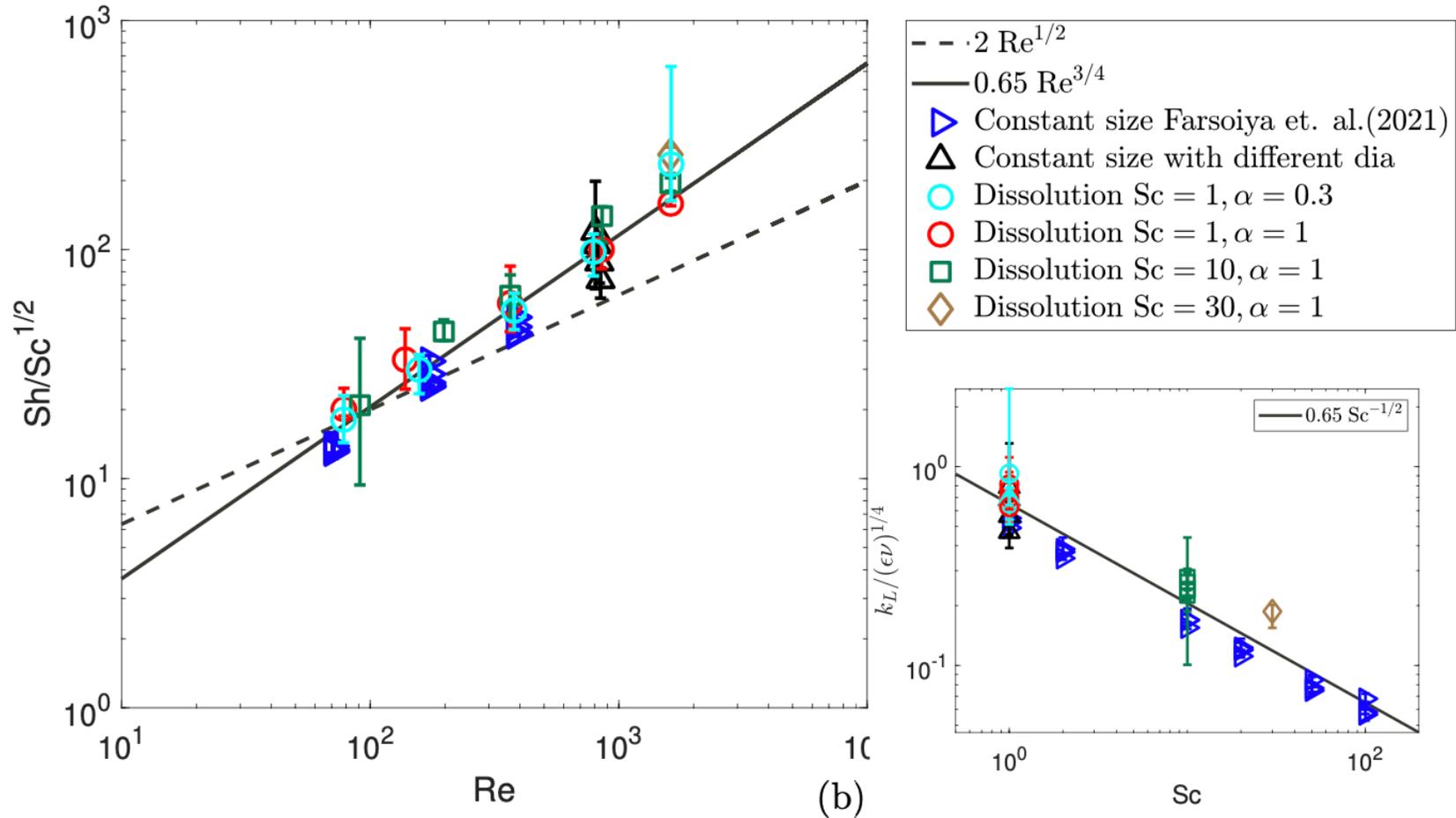
$$ShSc^{1/2} \propto Re^{3/4}$$

(see also Lamont and Scott 1970)



*Kumar Farsoiya, Popinet and Deike, J. Fluid Mech. 2021*  
*Kumar Farsoiya, Popinet and Deike, 2022, submitted*

# Mass (gas) transfer of an individual bubble in a turbulent flow



$$Sh/Sc^{1/2} \propto Re^{3/4}$$

$$k_L \propto Sc^{-1/2} (\epsilon \nu)^{1/4}$$

Same high Re regime as Herlina (Monday's talk)

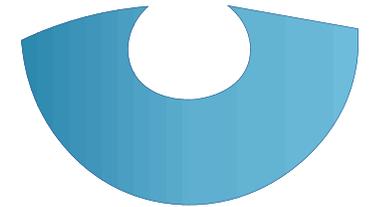
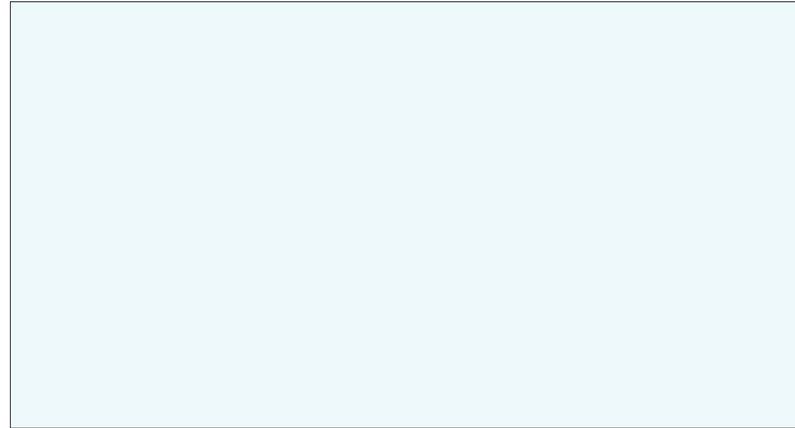
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# A multi-scale approach for ocean-atmosphere interaction

$O(1\text{m}-10\text{km})$

$O(1\text{mm}-10\text{m})$

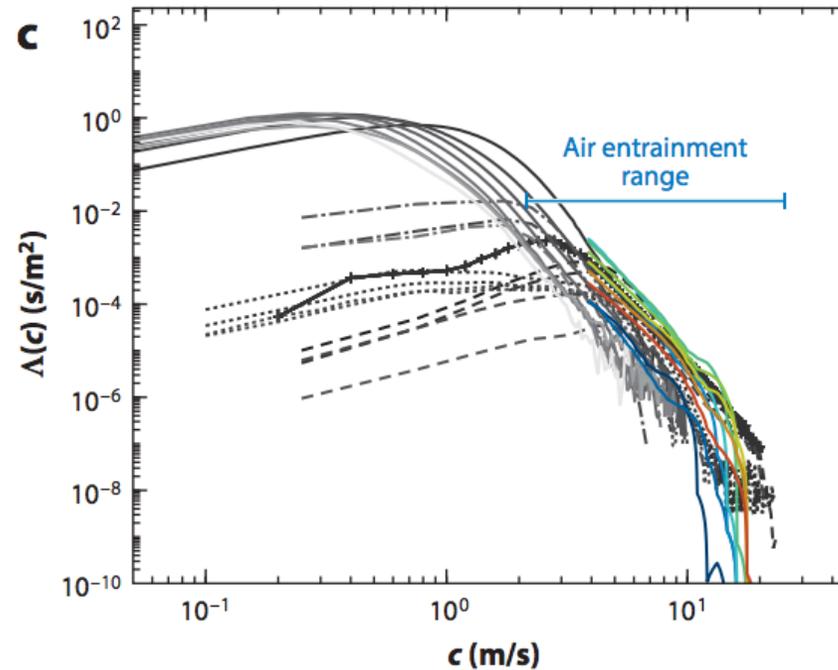
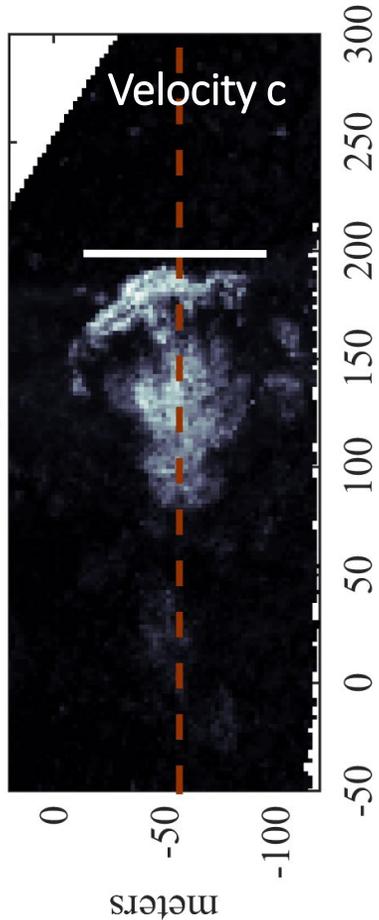
$O(1\mu\text{m}-10\text{mm})$



# Breaking wave distribution of length of breaking crest

## Breaking and air entrainment statistics

Visible video: detect and measure the length of breaking crest  $\Lambda(c)$ , moving at speed  $c$  and the associated breaking slope  $S=hk$  (via the wave spectrum) which control air entrainment (Phillips 1985, Melville et al 2016, Deike et al 2017)



- |                                  |                     |                       |                     |                       |
|----------------------------------|---------------------|-----------------------|---------------------|-----------------------|
| ..... Deike et al. (2017a)       | — $c_p/u_* = 0-10$  | — $c_p/u_* = 50-60$   | — $c_p/u_* = 0-10$  | — $c_p/u_* = 50-60$   |
| --- Sutherland & Melville (2013) | — $c_p/u_* = 10-20$ | — $c_p/u_* = 60-70$   | — $c_p/u_* = 10-20$ | — $c_p/u_* = 60-70$   |
| - - Kleiss & Melville (2010)     | — $c_p/u_* = 20-30$ | — $c_p/u_* = 70-100$  | — $c_p/u_* = 20-30$ | — $c_p/u_* = 70-100$  |
| -+ Schwendeman et al. (2014)     | — $c_p/u_* = 30-40$ | — $c_p/u_* = 100-170$ | — $c_p/u_* = 30-40$ | — $c_p/u_* = 100-170$ |
|                                  | — $c_p/u_* = 40-50$ |                       | — $c_p/u_* = 40-50$ |                       |

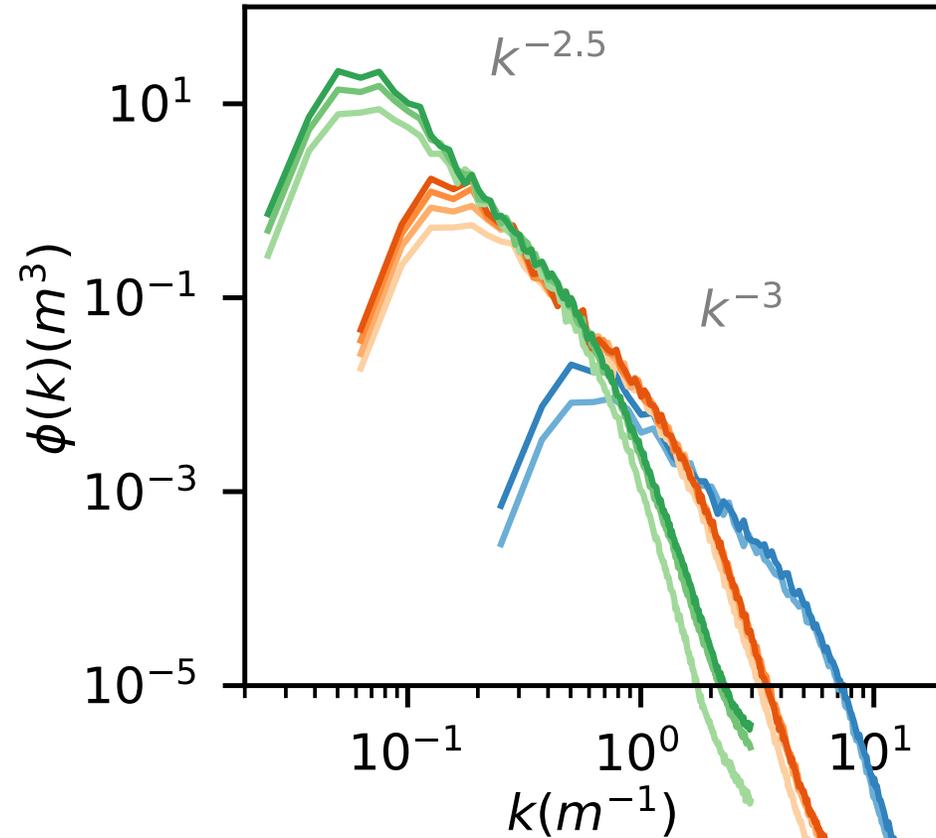
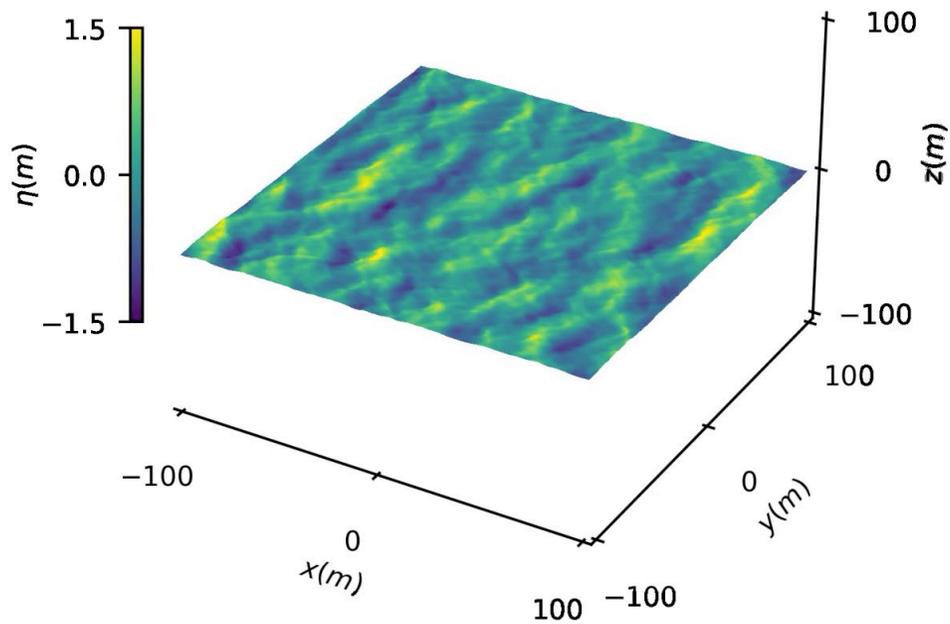
Airborne measurement of breaking waves, Melville et al 2016.

Deike et al 2017, Deike and Melville 2018; Deike 2022.

# Modeling wind waves and wave statistics



Direct modeling of a wave spectrum using the multi-layer model

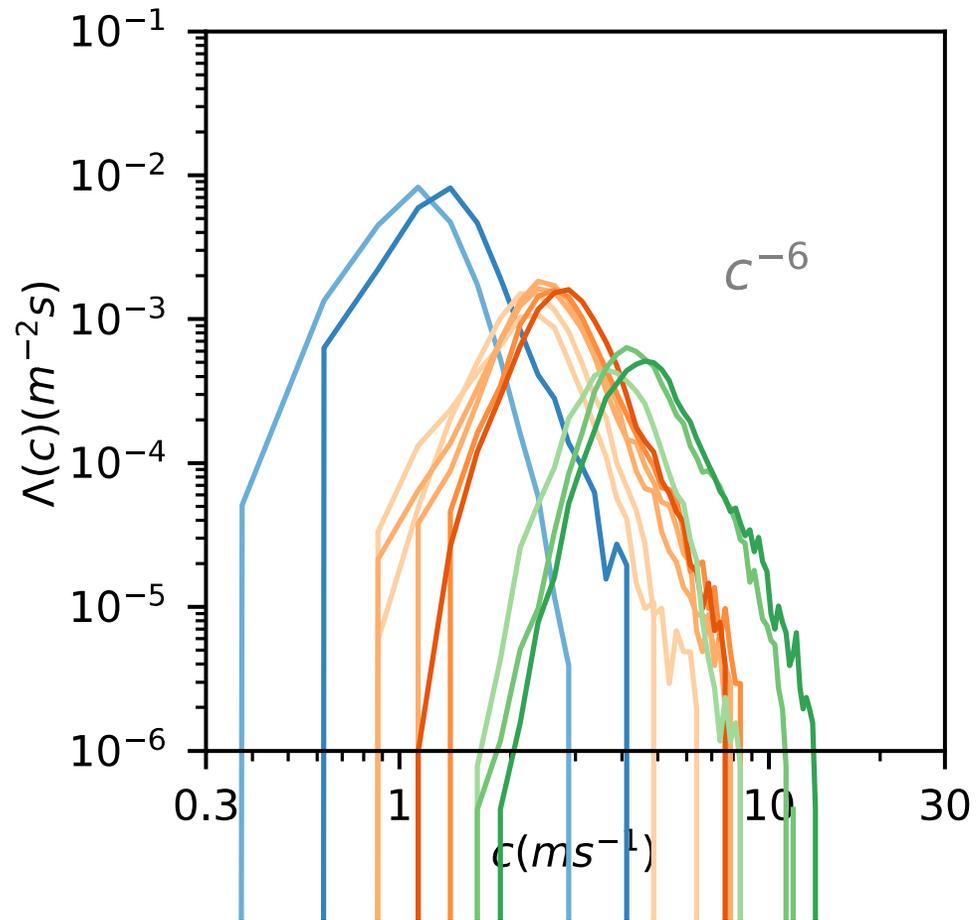


Wave spectrum:  $\phi(k) \propto Pk^{-5/2}$

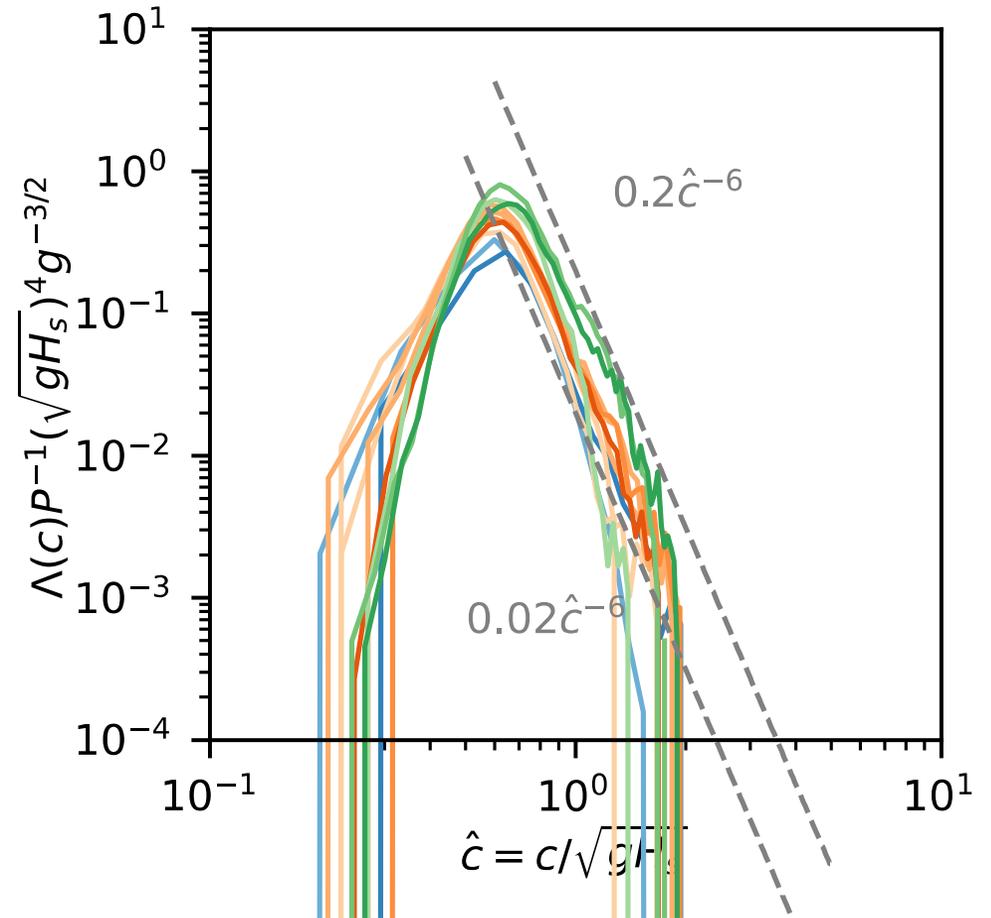
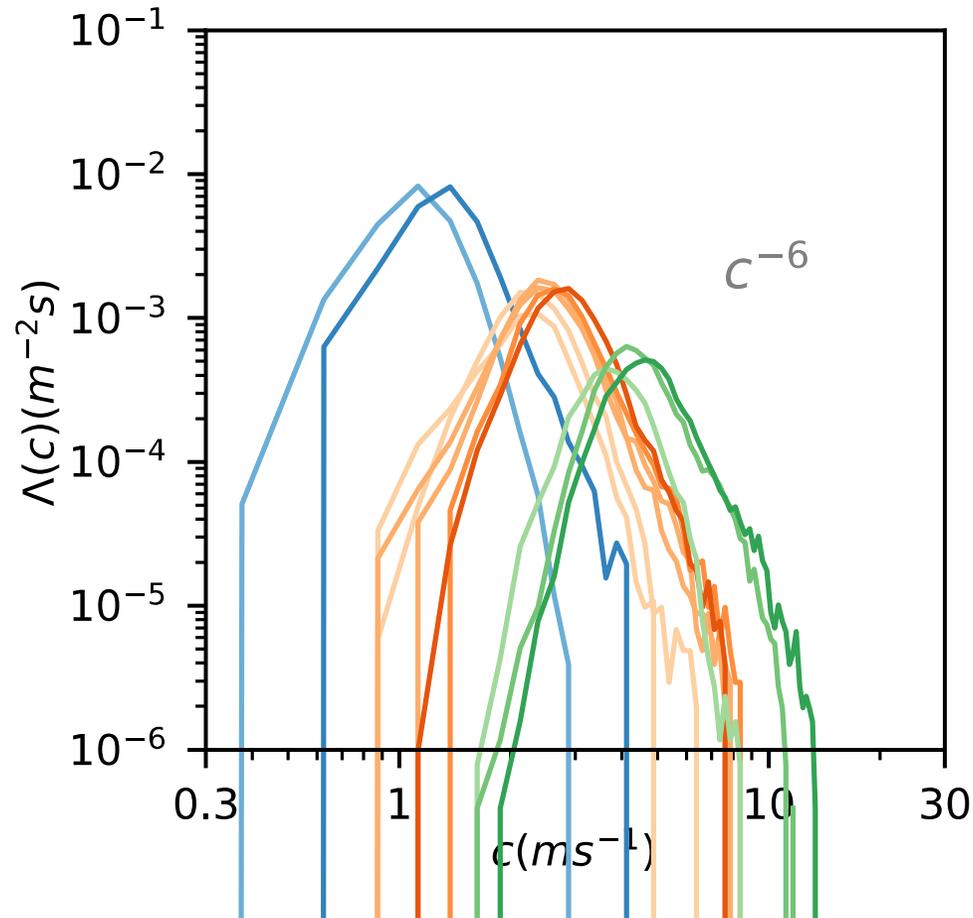
# Modeling wind waves and wave statistics



## Analysis of breaking kinematics statistics



# Scaling the breaking wave statistics



*Wu, Popinet and Deike, in prep*

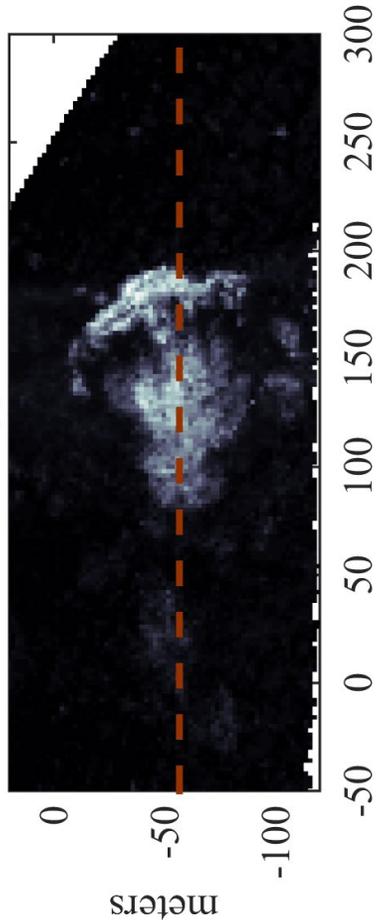
**Ability to model the breaking statistics based on the wave spectrum!**

(compatible to the spectral model from Romero 2019)

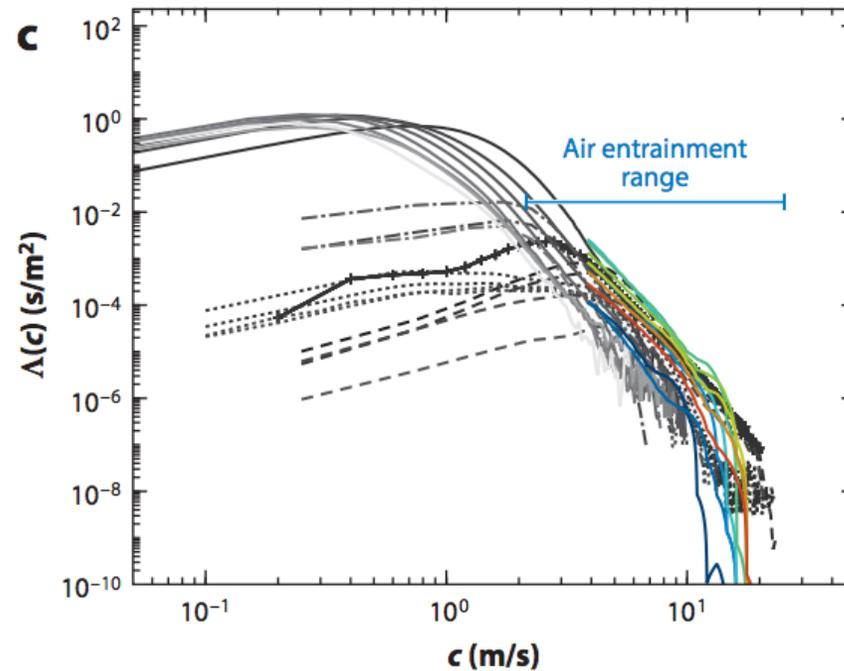
# Breaking wave distribution of length of breaking crest

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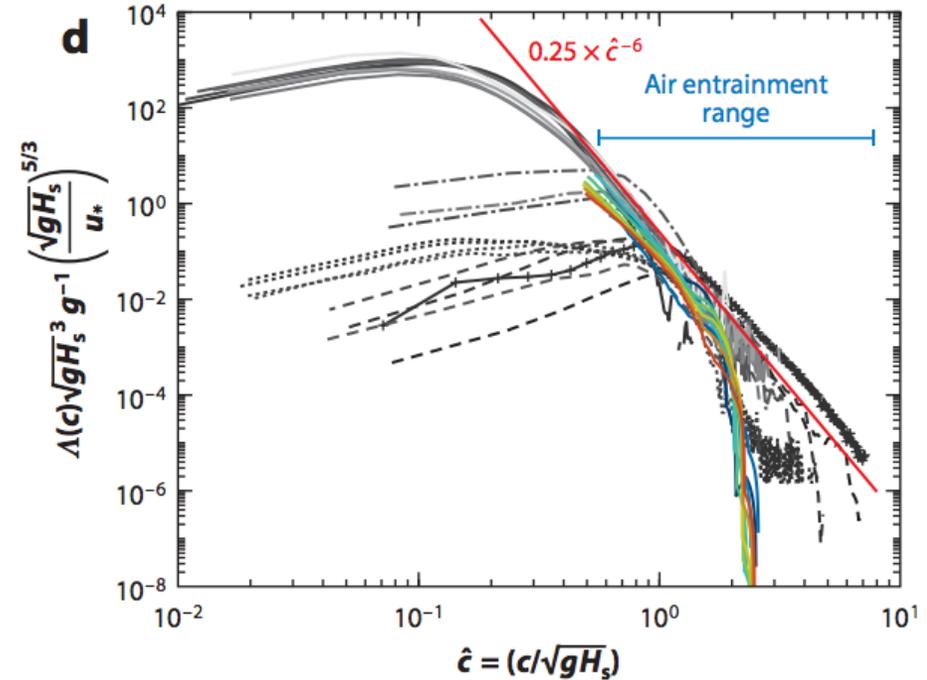
Airborne measurement of breaking waves, Melville et al 2016.



..... Deike et al. (2017a)  
 ---- Sutherland & Melville (2013)  
 - - - Kleiss & Melville (2010)  
 -+ Schwendeman et al. (2014)

### Sutherland & Melville (2013)

—  $c_p/u_* = 0-10$     —  $c_p/u_* = 50-60$   
 —  $c_p/u_* = 10-20$    —  $c_p/u_* = 60-70$   
 —  $c_p/u_* = 20-30$    —  $c_p/u_* = 70-100$   
 —  $c_p/u_* = 30-40$    —  $c_p/u_* = 100-170$   
 —  $c_p/u_* = 40-50$



### Romero (2019)

—  $c_p/u_* = 0-10$     —  $c_p/u_* = 50-60$   
 —  $c_p/u_* = 10-20$    —  $c_p/u_* = 60-70$   
 —  $c_p/u_* = 20-30$    —  $c_p/u_* = 70-100$   
 —  $c_p/u_* = 30-40$    —  $c_p/u_* = 100-170$   
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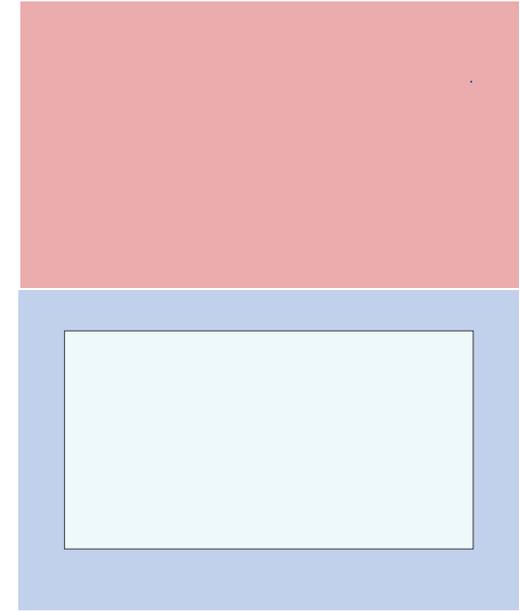
Deike et al 2017, Deike and Melville 2018; Deike 2022.

# Air entrainment by an ensemble of breaking waves

$$V_A = \int v_l(c) \Lambda(c) dc, \text{ with } v_l(c) = V / (L_c \tau_b),$$

Entrained air by a breaking wave with speed  $c$ , slope  $hk$

Integration over all breaking events  
(*Deike et al 2016, 2017*); follows *Phillips 1985*



# Air entrainment by an ensemble of breaking waves

$$V_A = \int v_l(c) \Lambda(c) dc, \text{ with } v_l(c) = V / (L_c \tau_b),$$

Entrained air by a breaking wave with speed  $c$ , slope  $hk$

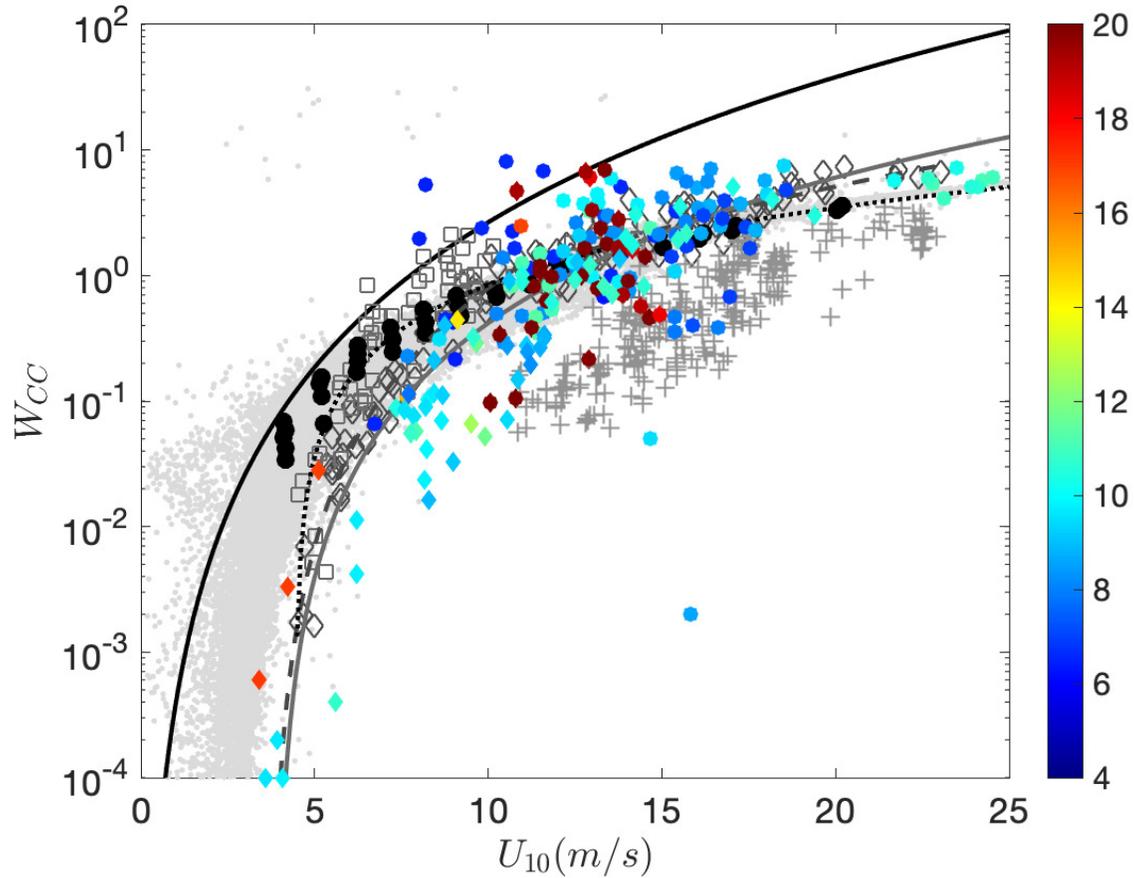
Integration over all breaking events  
(Deike et al 2016, 2017); follows Phillips 1985

$$V_A = \int B \frac{b}{(hk)} \frac{c^3}{g} \Lambda(c) dc.$$

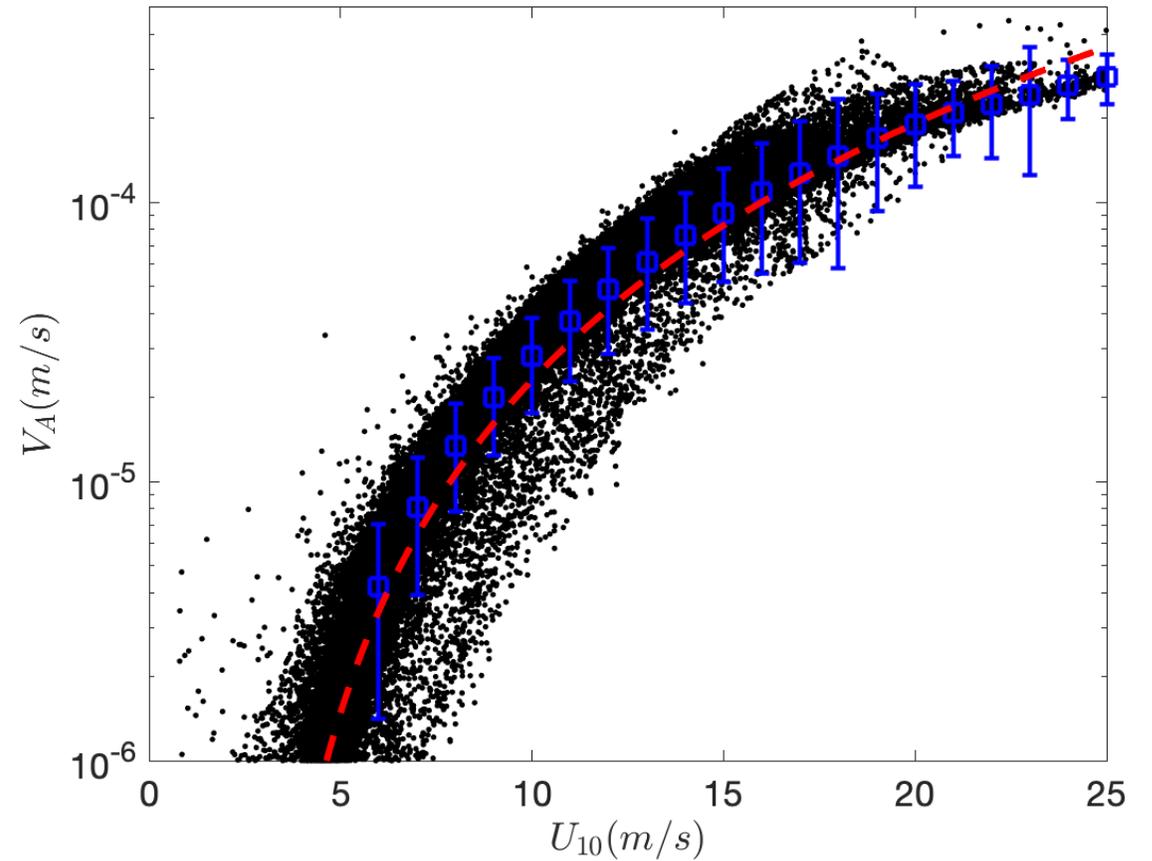
Similarly the size distribution of bubbles under an ensemble of breaking waves

$$Q(R_b) = \int q_l(R_b, c) \Lambda(c) dc, \text{ with } q_l(R_b, c) = \frac{B}{2\pi} s(k)^{3/2} N(R_b) \frac{c^3}{g},$$

# Air entrainment and whitecap coverage



$$W_{CC} \propto \int c^2 \Lambda(c) dc$$



$$V_A \propto \int c^3 \Lambda(c) dc$$

# A multi-scale model for the total gas flux

$$F = -(k_b + k_{nb})\Delta C$$

*Woolf and Thorpe 1991*  
*Keeling 1993,*

Non-breaking transfer, from classic parameterization  
(Edson et al 2011, Zappa et al 2007):

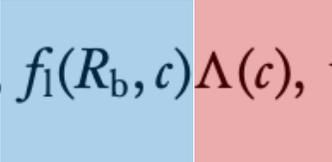
$$k_{nb} = ASc^{-1/2}u_*$$

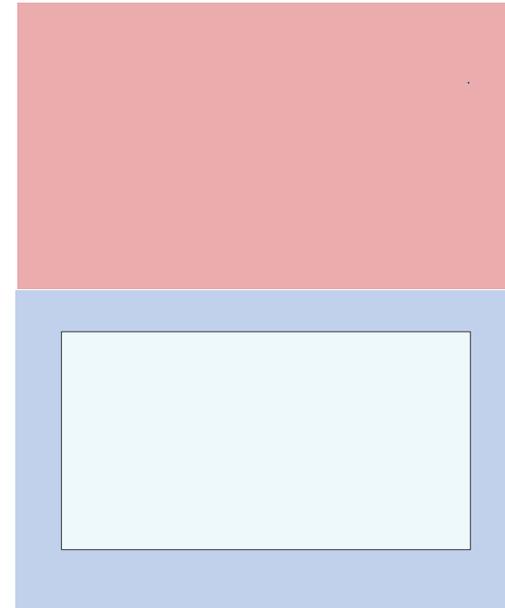
# A multi-scale model for the total gas flux

$$F = -(k_b + k_{nb})\Delta C$$

*Woolf and Thorpe 1991*  
*Keeling 1993,*

Bubble mediated flux  $k_b$ , **using individual bubble dynamics in turbulence results for rise velocity and transfer rates, integrated over the bubble size distribution and all breaking events**

$$k_b = \iint dc dR_b f_l(R_b, c) \Lambda(c), \text{ with}$$




*Deike and Melville 2018;*

# A multi-scale model for the total gas flux

$$F = -(k_b + k_{nb})\Delta C$$

Bubble mediated flux  $k_b$ , **using individual bubble dynamics in turbulence results for rise velocity and transfer rates, integrated over the bubble size distribution and all breaking events**

$$k_b = \iint dc dR_b f_1(R_b, c) \Lambda(c), \text{ with}$$

$$f_1(R_b, c) = \frac{q_1(R_b, c) E(R_b)}{\alpha} \frac{4\pi R_b^3}{3} = \frac{B}{2\pi} \frac{s(k)^{3/2} c^3}{g} \frac{4\pi R_b^3}{3} \frac{N(R_b) E(R_b)}{\alpha}$$

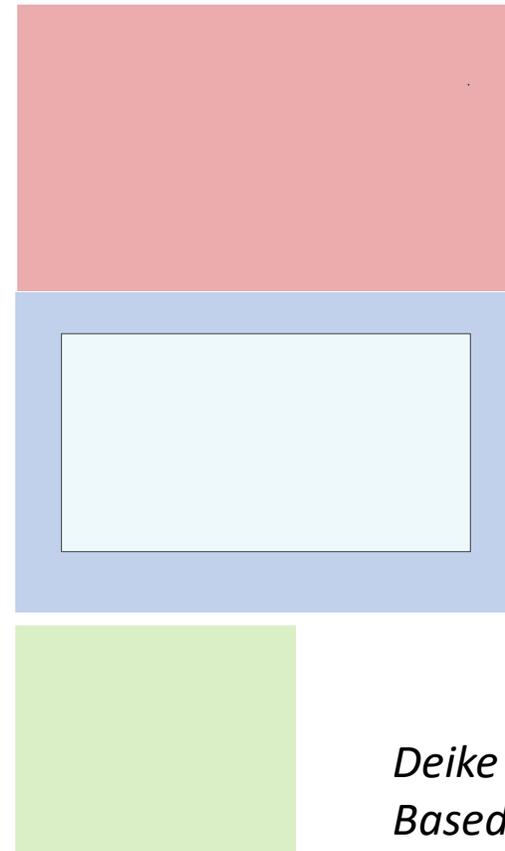
With:

$$E(R_b) = \frac{z_0}{z_0 + He_q}$$

Efficiency factor

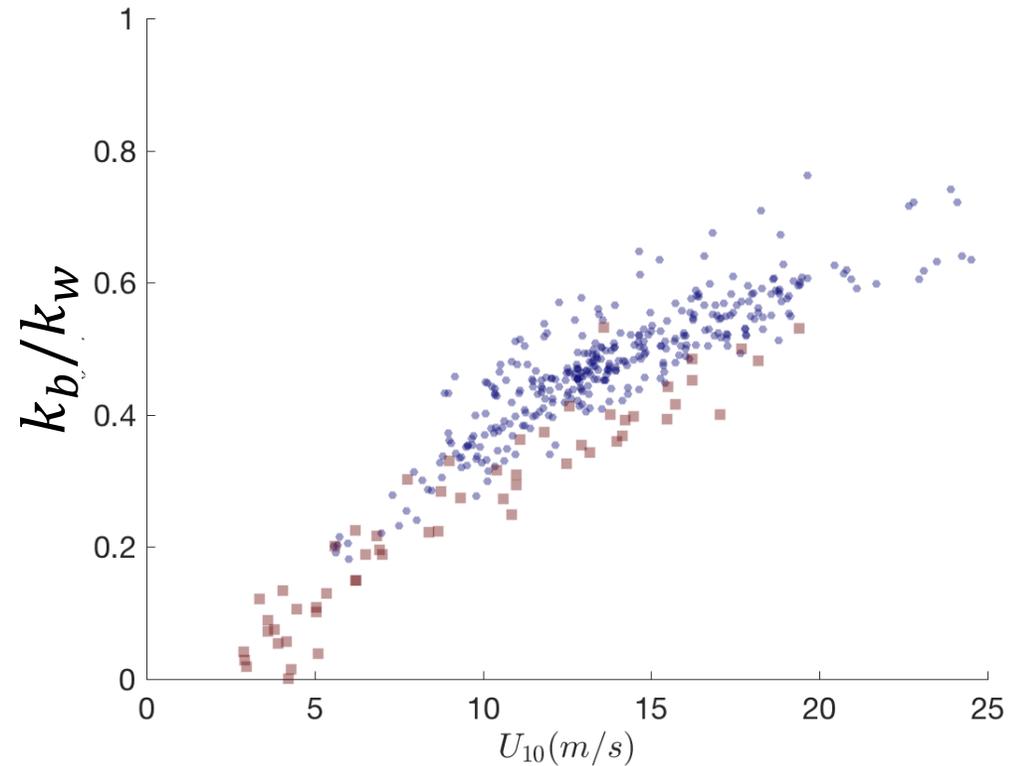
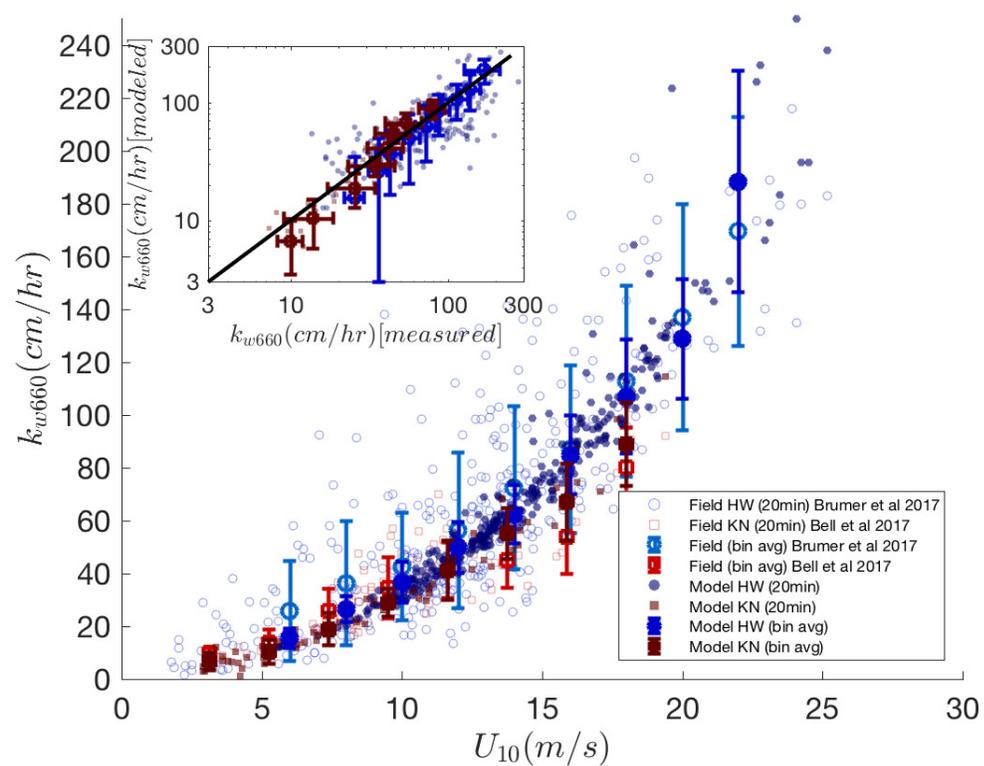
$$H_{eq} = \frac{4\pi}{3\alpha} \frac{R_b w_b}{k_L}$$

Equilibrium depth



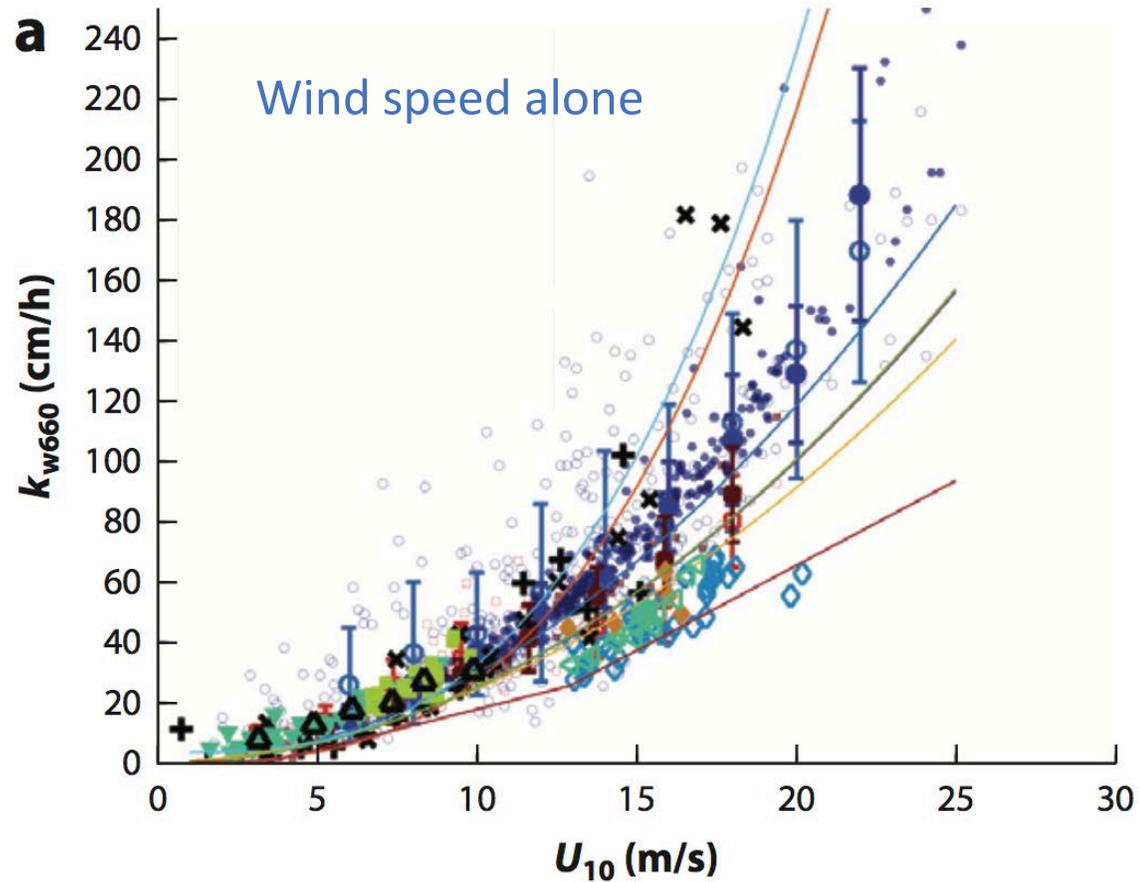
*Deike and Melville 2018;  
Based on Keeling 1993*

# The model reproduces recent CO<sub>2</sub> gas transfer velocity field measurements



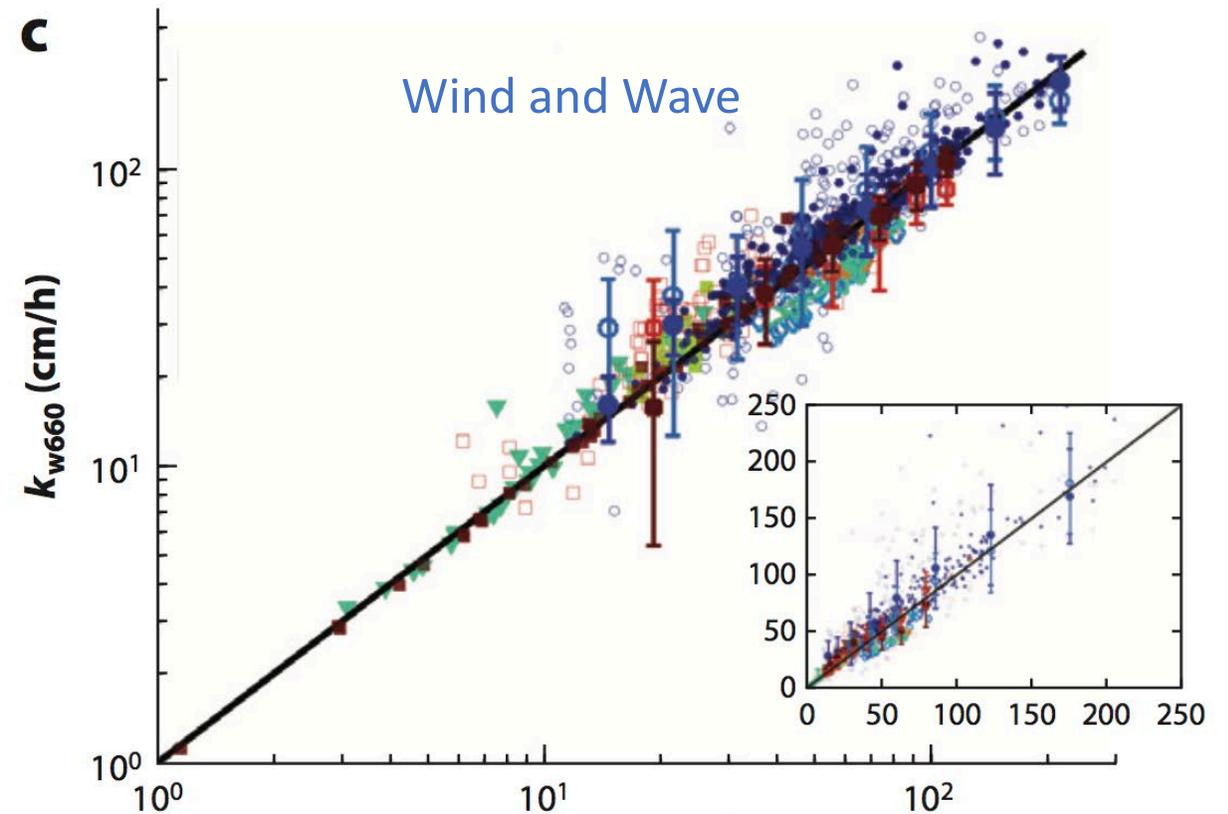
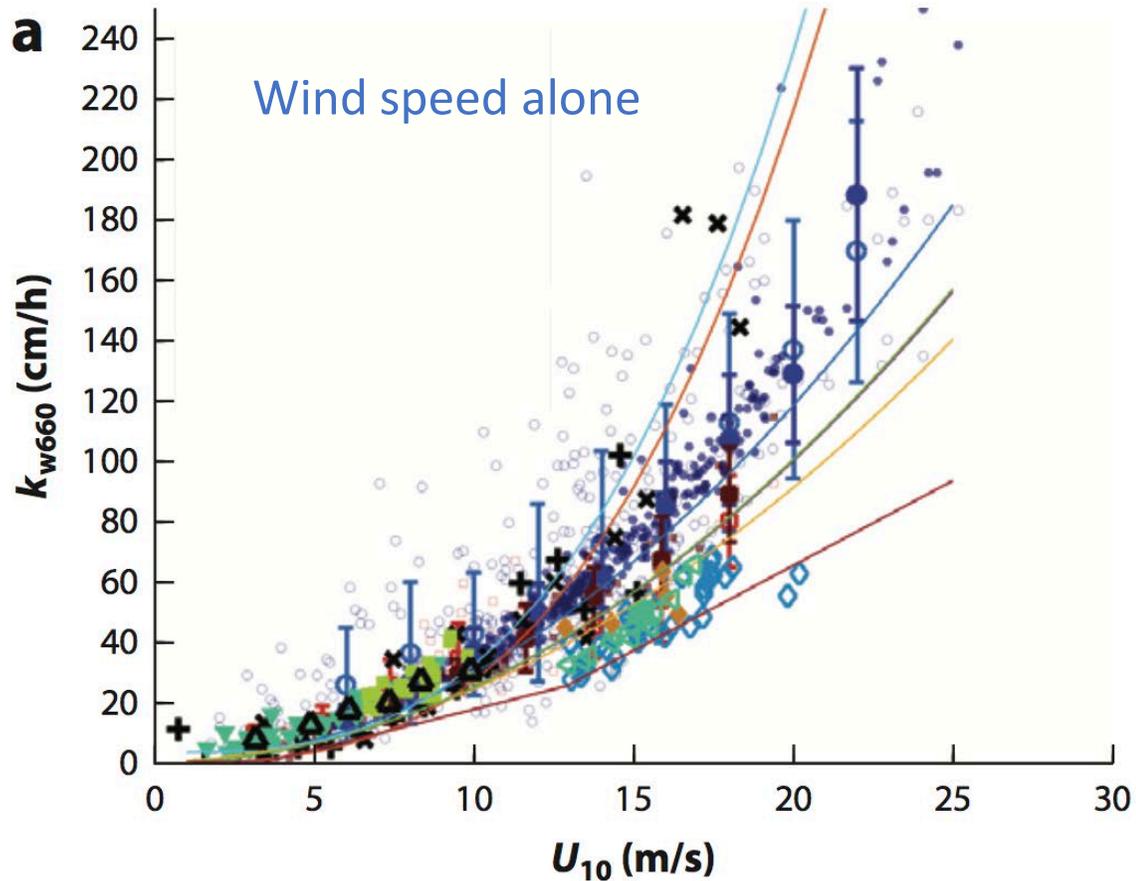
Data sets in the North Atlantic, *Brumer et al 2017* (HiWings), *Bell et al 2017* (Knorr 2011)  
Bubble mediated gas transfer becomes dominant above 15 m/s

# Gas transfer velocity: a function of wind speed and sea-state



- Field: HW (20 min) (Brumer et al. 2017a)
- Field: KN (20 min) (Bell et al. 2017)
- ⊕ Field: bin avg. (Brumer et al. 2017a)
- ⊗ Field: bin avg. (Bell et al. 2017)
- Model: HW (20 min)
- Model: KN (20 min)
- ⊕ Model: HW (bin avg.)
- ⊗ Model: KN (bin avg.)
- + Field: GasEx 98 (Edson et al. (2011))
- × Field: SO GasEx (Edson et al. (2011))
- △ Field: Knorr 07 (Miller et al. (2011))
- ◇ Model: VI Gotex
- ◁ Model: VI Hires 2010
- ▽ Model: IR Socal 2010
- ◆ Model: Hires 2010
- Model: Radyo 2009
- 1:1

# Gas transfer velocity: a function of wind speed and sea-state



$$k_{nb} + k_b = A_{nb} u_* + \frac{A_b}{\alpha} u_*^{5/3} (\sqrt{gH_s})^{4/3}$$

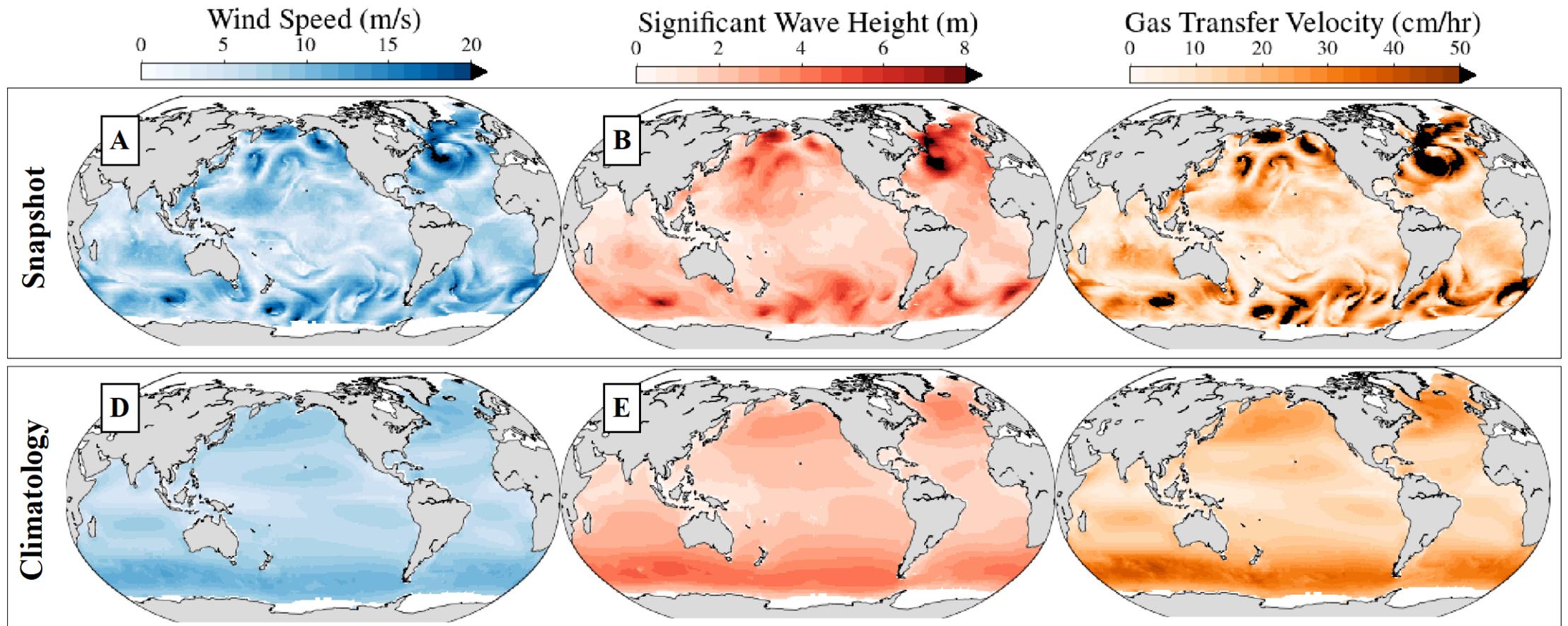
- |  |                        |   |                        |       |
|--|------------------------|---|------------------------|-------|
| ○ Field: HW (20 min) (Brumer et al. 2017a) | ● Model: HW (20 min)   | + Field: GasEx 98 (Edson et al. (2011)) | ◁ Model: VI Hires 2010 | — 1:1 |
| □ Field: KN (20 min) (Bell et al. 2017)    | ■ Model: KN (20 min)   | × Field: SO GasEx (Edson et al. (2011)) | ▽ Model: IR Socal 2010 |       |
| ◻ Field: bin avg. (Brumer et al. 2017a)    | ◼ Model: HW (bin avg.) | △ Field: Knorr 07 (Miller et al. 2011)  | ◇ Model: Hires 2010    |       |
| ◻ Field: bin avg. (Bell et al. 2017)       | ◼ Model: KN (bin avg.) | ◇ Model: VI Gotex                       | ■ Model: Radyo 2009    |       |

*Deike and Melville 2018*

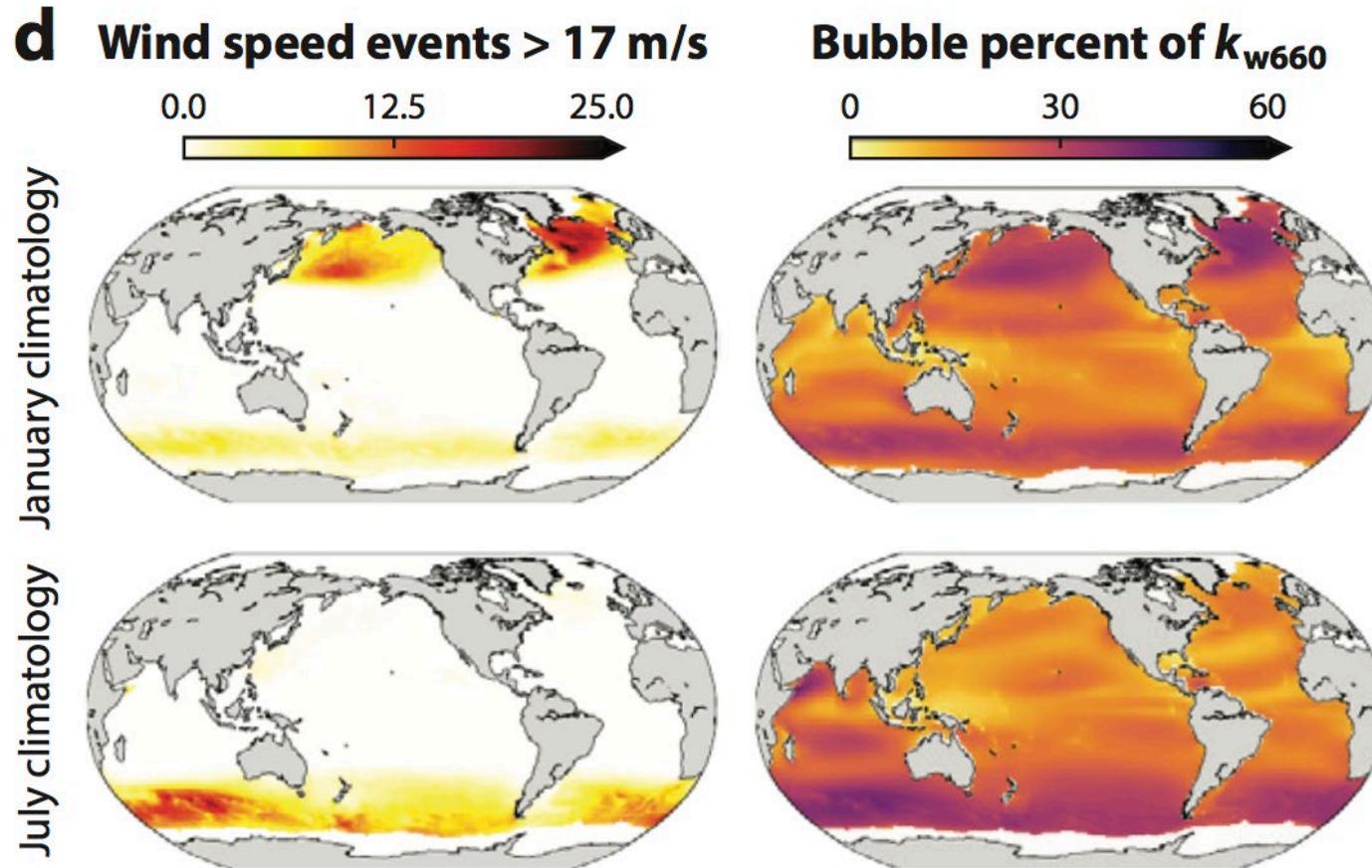
Semi-empirical parameterization inspired from our theoretical model collapses all data for CO<sub>2</sub> (and DMS)

$$k_{w,660} = k_{w,660} \left( \frac{Sc}{660} \right)^{1/2} = k_{nb} + k_b = A_{nb} u_* + \frac{A_b}{\alpha} u_*^{5/3} (\sqrt{gH_s})^{4/3}$$

# Implementation of our wind-wave gas transfer parameterization: Global wave modeling (with Brandon Reichl, NOAA GFDL)



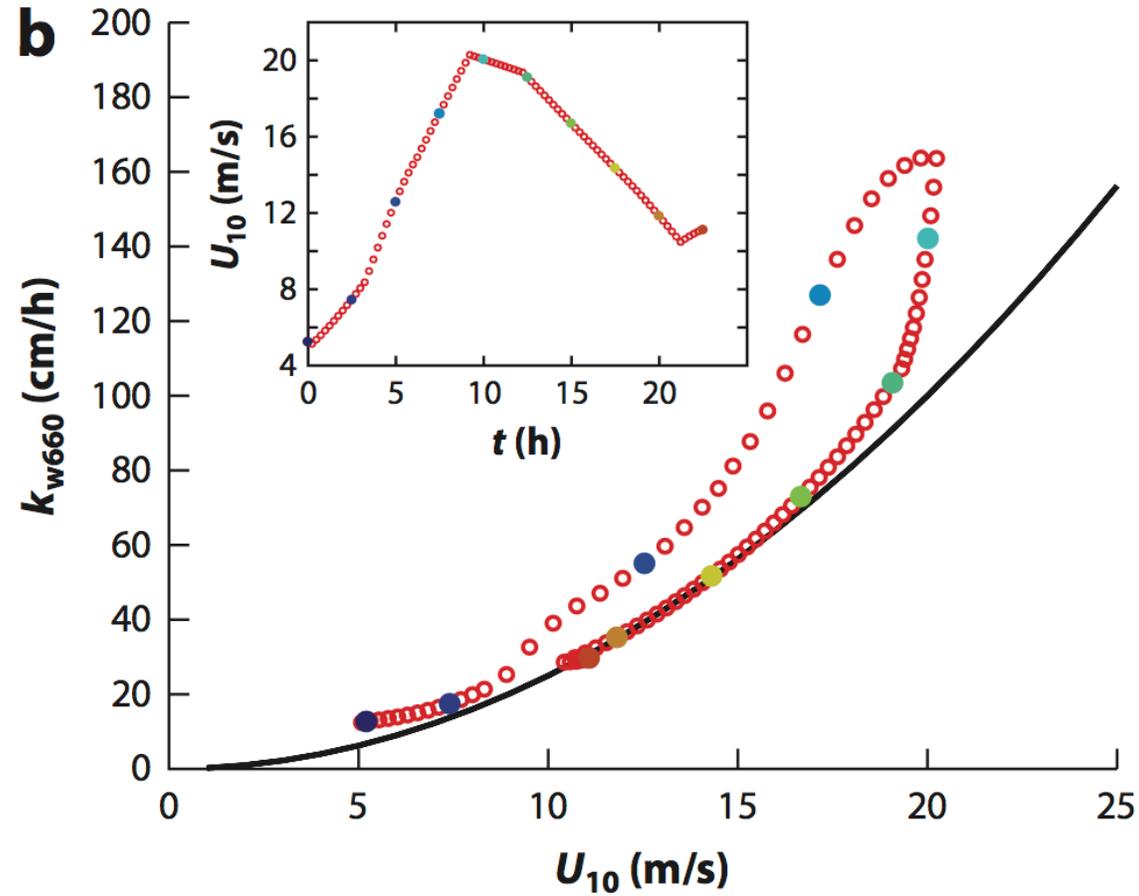
# Understanding the role of sea-state variability during storms and its implications on regional and seasonal scales



Bubble accounts for 40% of the total CO<sub>2</sub> flux and increase the spatial and temporal flux variability

*Reichl and Deike 2020*

# Understanding the role of sea-state variability during storms and its implications on regional and seasonal scales



Transfer velocity during storm intensification can be twice larger for the same wind speed value

Hysteresis cycles are sometimes the other way around

# Conclusions and Perspectives

Multi-scale framework to describe air-sea mass exchange related to bubbles

From air entrainment, bubble distribution under breaking waves, to gas transfer under an ensemble of breaking waves

Can be applied locally and globally using wave modeling

Describes induced sea-state variability

**Need to incorporate bubble collapse term (bubble asymmetric effect)**

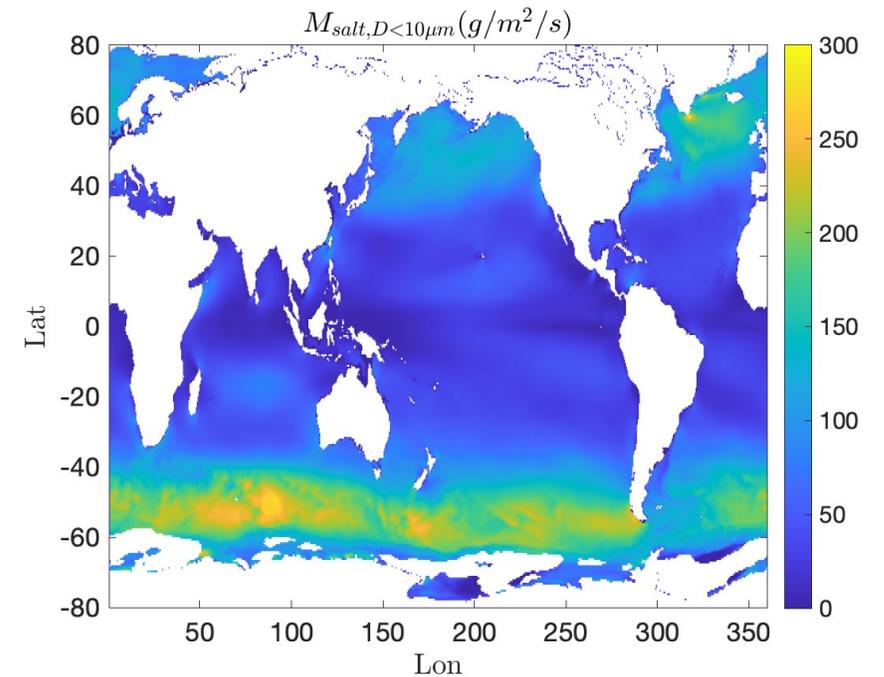
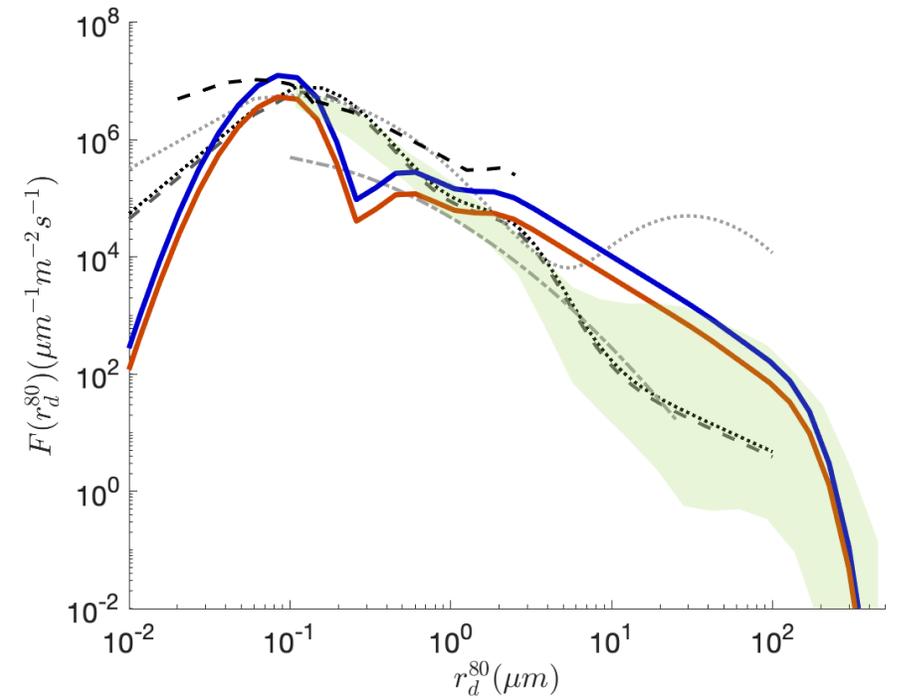
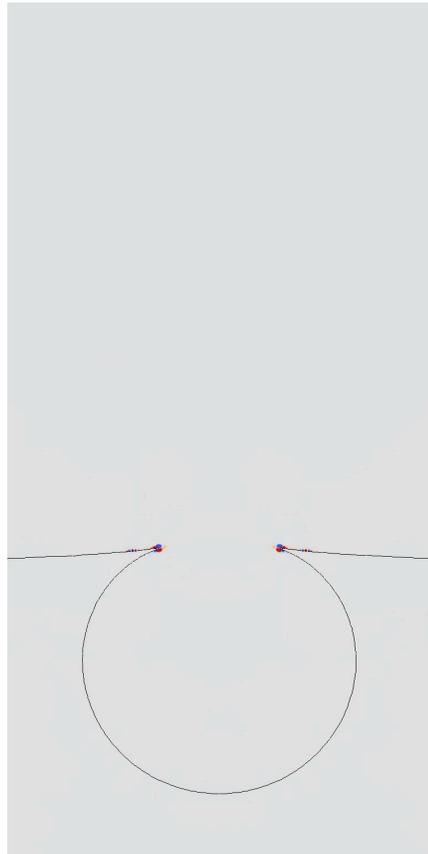
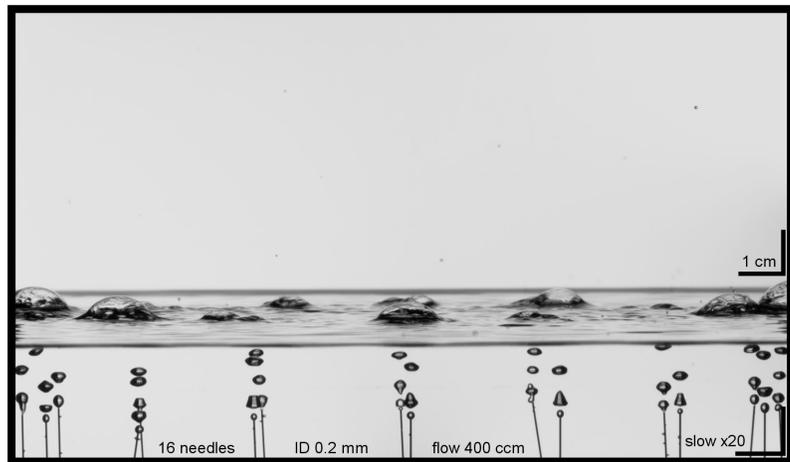
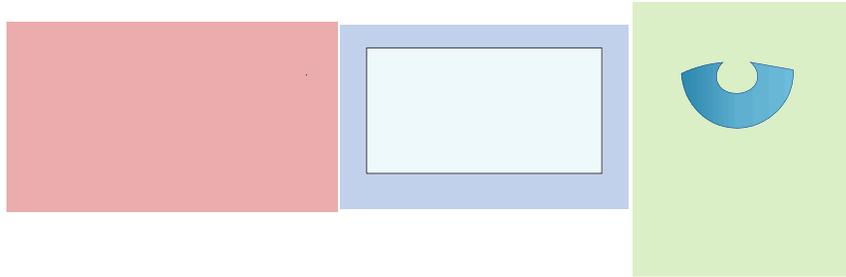
**If you have gas fluxes/transfer velocity data, I would be happy to see if the present framework can model them!**

Similar ideas are being applied to sea spray production by bubble bursting

# Sea spray by bubble bursting!

$$F_d(r_d) = \int Q(R_b) \frac{n(R_b)}{\langle r_d \rangle} p\left(\frac{r_d}{\langle r_d \rangle}\right) dR_b$$

$$= \iint \frac{B}{2\pi} \frac{s(k)^{3/2} c^3}{g} \Lambda(c) dc \frac{N(R_b) n(R_b)}{\langle r_d \rangle} p\left(\frac{r_d}{\langle r_d \rangle}\right) dR_b.$$



# Conclusions and Perspectives

## **Multi-scale framework to describe air-sea mass exchange related to bubbles**

Deike et al 2017 GRL, Deike and Melville 2018 GRL, Deike 2022 Ann. Rev. Fluid Mech

## **Describe air entrainment, bubble statistics and turbulence under breaking waves**

(Mostert, Popinet and Deike, 2022, in press, Deike et al 2016, J. Fluid Mech.)

## **Bubble size distribution in turbulence can be explained by local and non-local production processes**

(Riviere, Mostert, Perrard and Deike, 2021, Riviere, Ruth, Mostert, Deike and Perrard, 2022 under review.)

## **Individual bubble rise velocity and gas transfer in turbulence and under breaking waves**

P. Kumar Farsoiya, S. Popinet and L. Deike (2021). JFM. D.J. Ruth, M. Vernet, S. Perrard and L. Deike (2021), JFM. And work in progress

## **Application to global models using spectral wave modeling**

B. Reichl and L. Deike (2020). GRL, 47, 9. And work in progress

## **Sea spray production by bubble bursting: jet drop production**

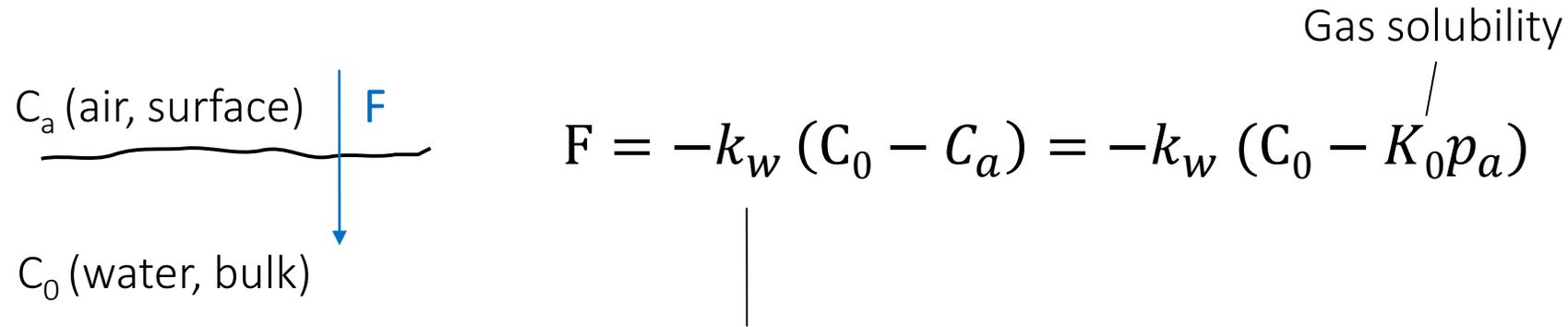
Deike et al 2018 PRF, Lai, Eggers and Deike 2018 PRL, Berny, Deike, Seon, Popinet 2020, 2021 PRF, GRL;

## **Towards understanding collective processes, role of contamination and temperature**

Neel and Deike 2021 JFM, Shaw and Deike 2021 JFM, and work in progress

## **Wind wave modeling (J. Wu and L. Deike 2021 (PRF), and Wu et al submitted)**

# How is gas transfer modeled?



Wanninkhov et al 2009  
Garbe et al 2014

$$k_w \propto Sc^{-n} U_{10}^m$$

Schmidt number  
 $Sc = \nu/D$

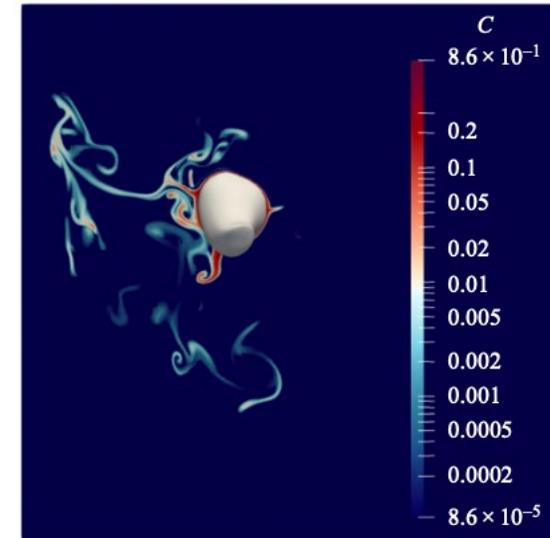
Wind speed  $U_{10}$

Is the wind speed good enough?

# Mass (gas) transfer of an individual bubble in a turbulent flow



Exchange of a dilute gas and dissolution through a one fluid advection equation based on Bothe & Fleckenstein 2013, Haroon et al 2010.



*Kumar Farsoiya, Popinet and Deike, J. Fluid Mech. 2021*  
*Kumar Farsoiya, Popinet and Deike, 2022, submitted*

# Mass (gas) transfer of an individual bubble in a turbulent flow



Higbie' 1938 eddy renewal theory:  $k_L \propto \sqrt{\frac{D}{\theta}}$

For a bubble/interface in a turbulent flow:  $\theta \propto \frac{\eta_K}{u'_k}$

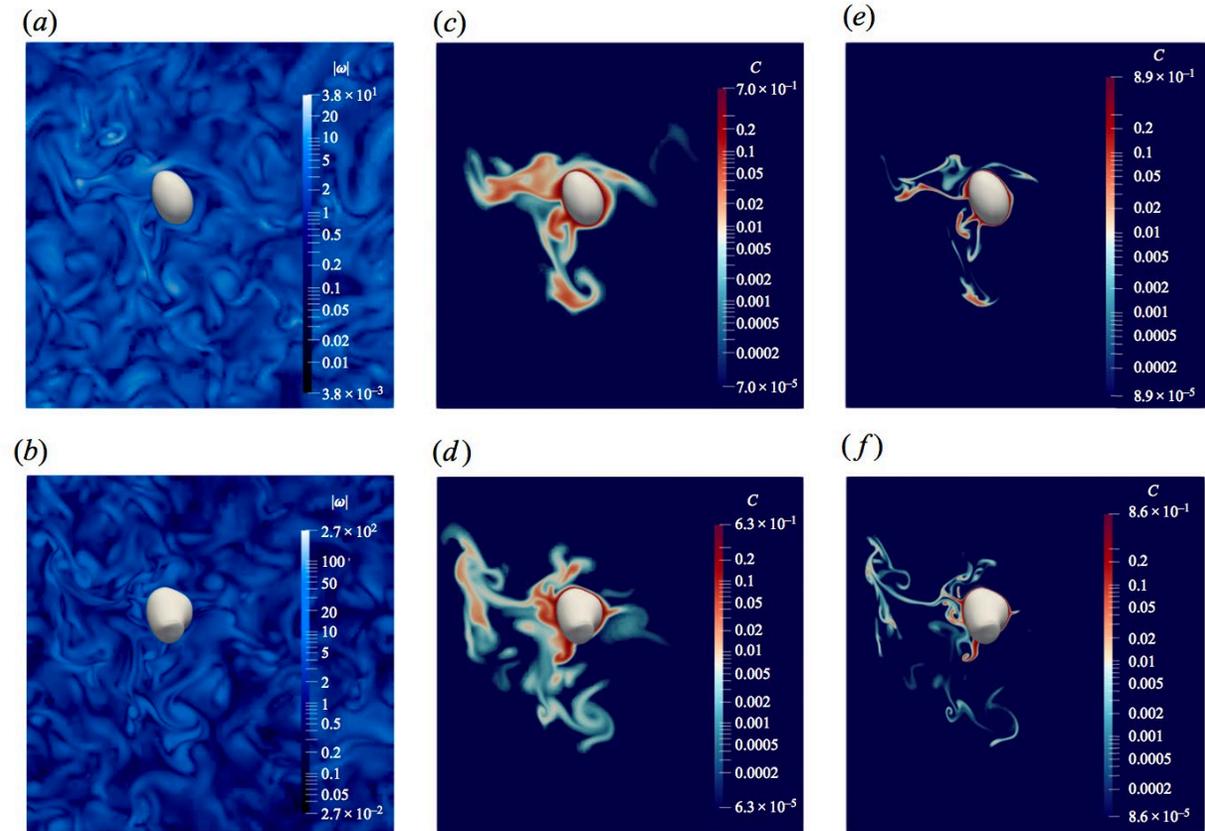
The transfer rate then reads:

$$k_L \propto Sc^{-1/2} (\varepsilon \nu)^{1/4}$$

In non dimensional form

$$ShSc^{1/2} \propto Re^{3/4}$$

(see also Lamont and Scott 1970)



Vorticity

$Sc=1$  ( $Pe=200$ )

$Sc=10$  ( $Pe=2000$ )

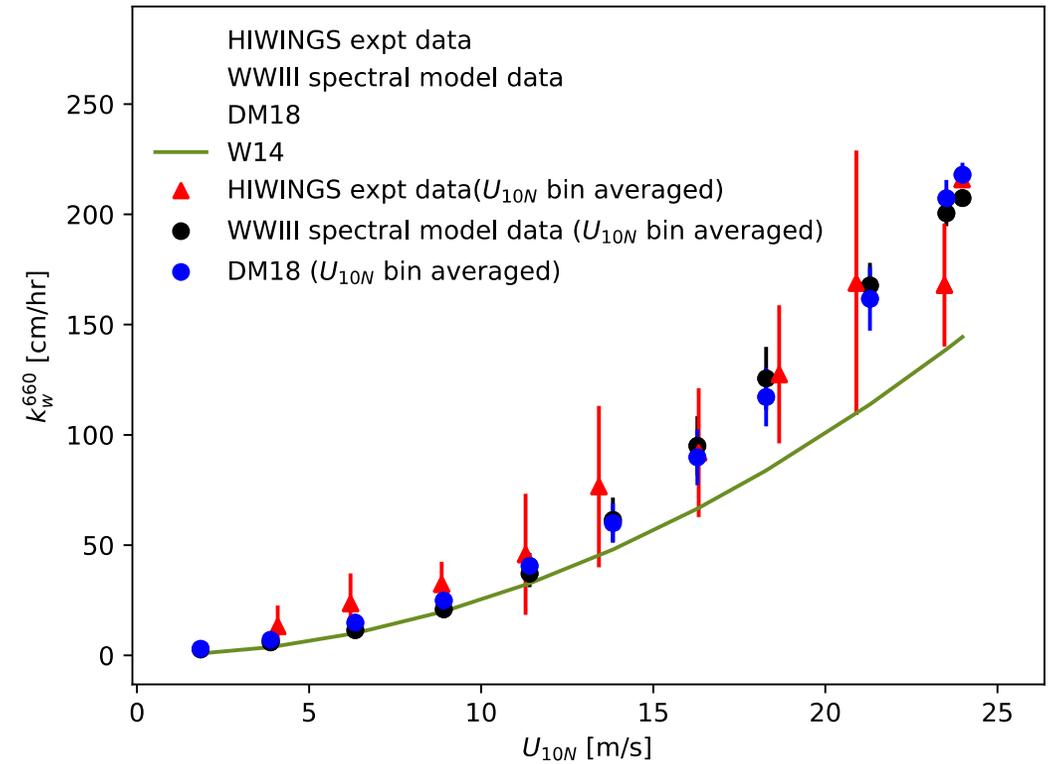
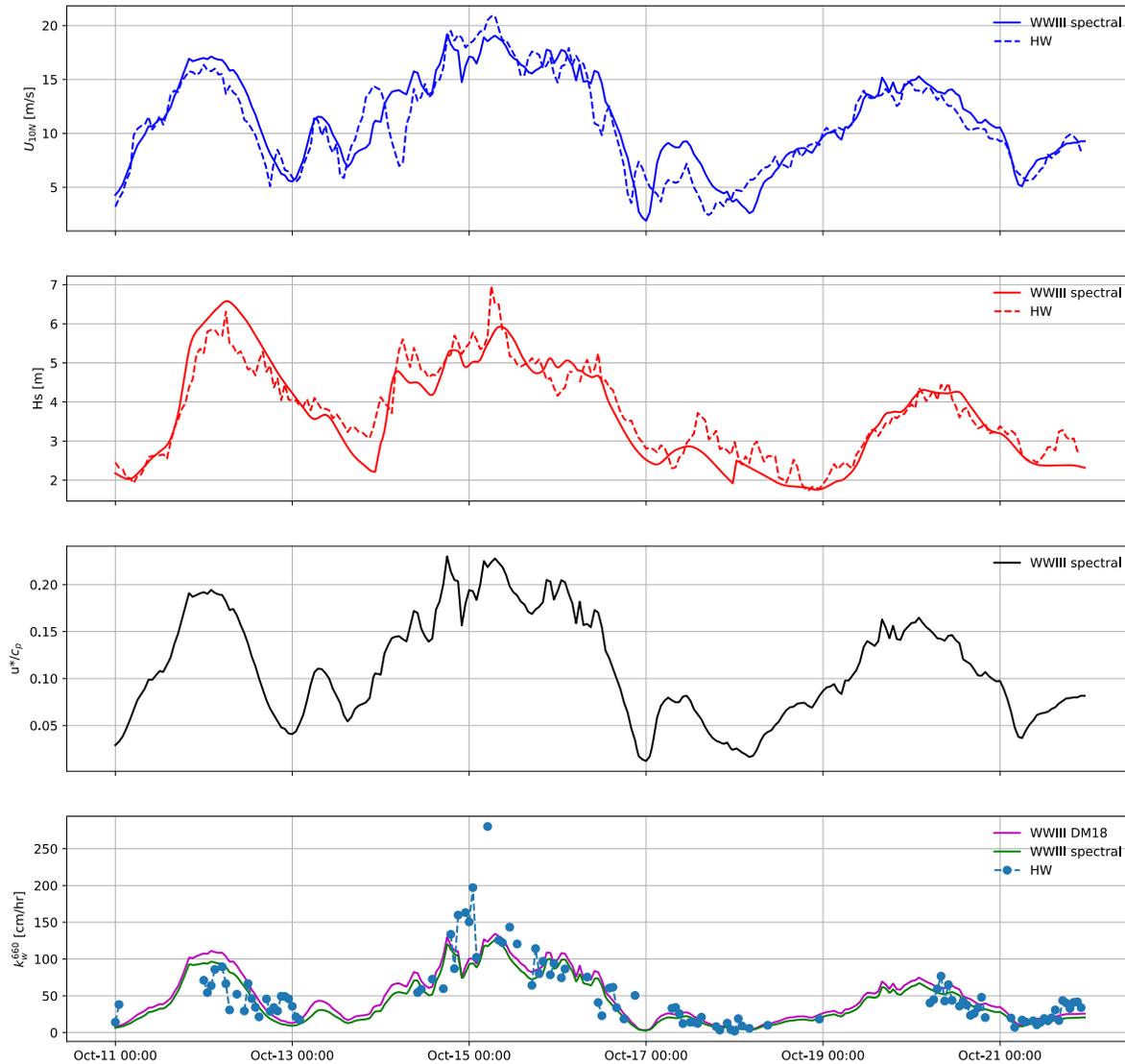
Kumar Farsoiyya, Popinet and Deike, *J. Fluid Mech.* 2021

Kumar Farsoiyya, Popinet and Deike, 2022, submitted

# The model reproduces recent CO<sub>2</sub> k<sub>w</sub> field measurements



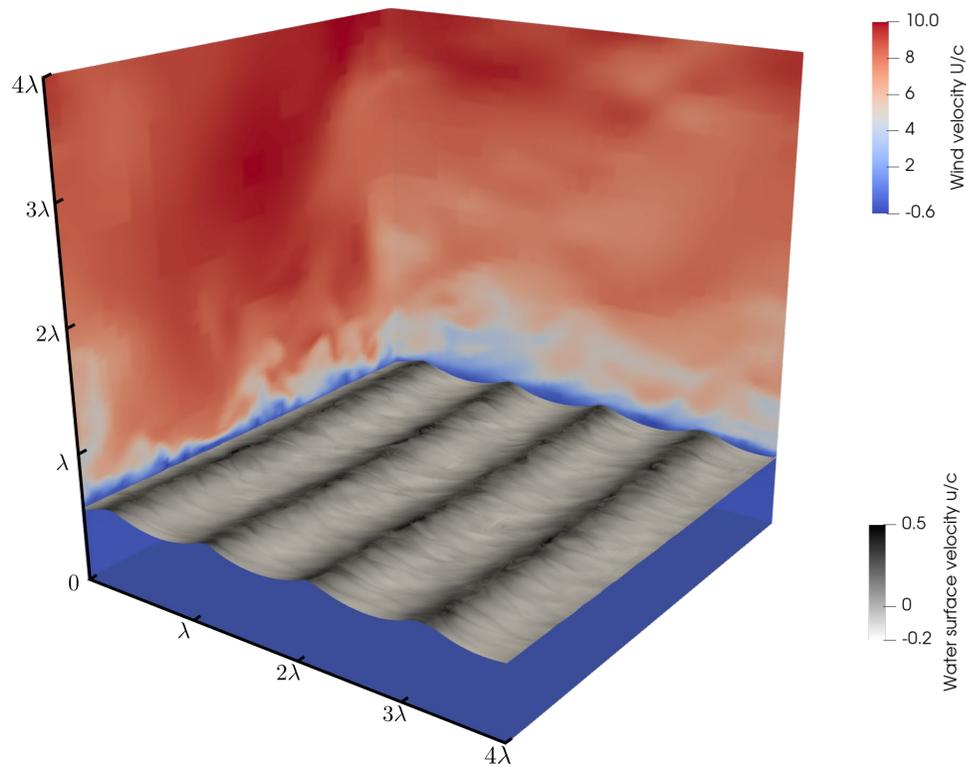
Comparing WWIII model against HiWinGS data



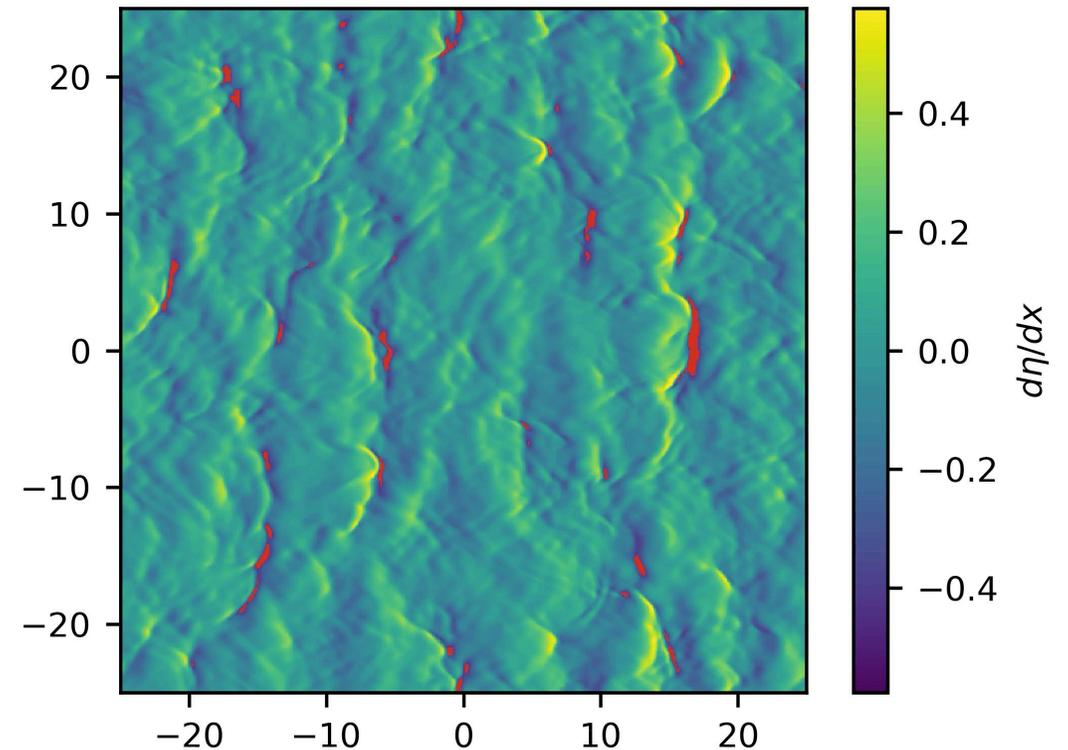
# Modeling wind waves and wave statistics



Wind wave growth via full DNS coupling  
(Wu and Deike 2021, and Wu et al, submitted)

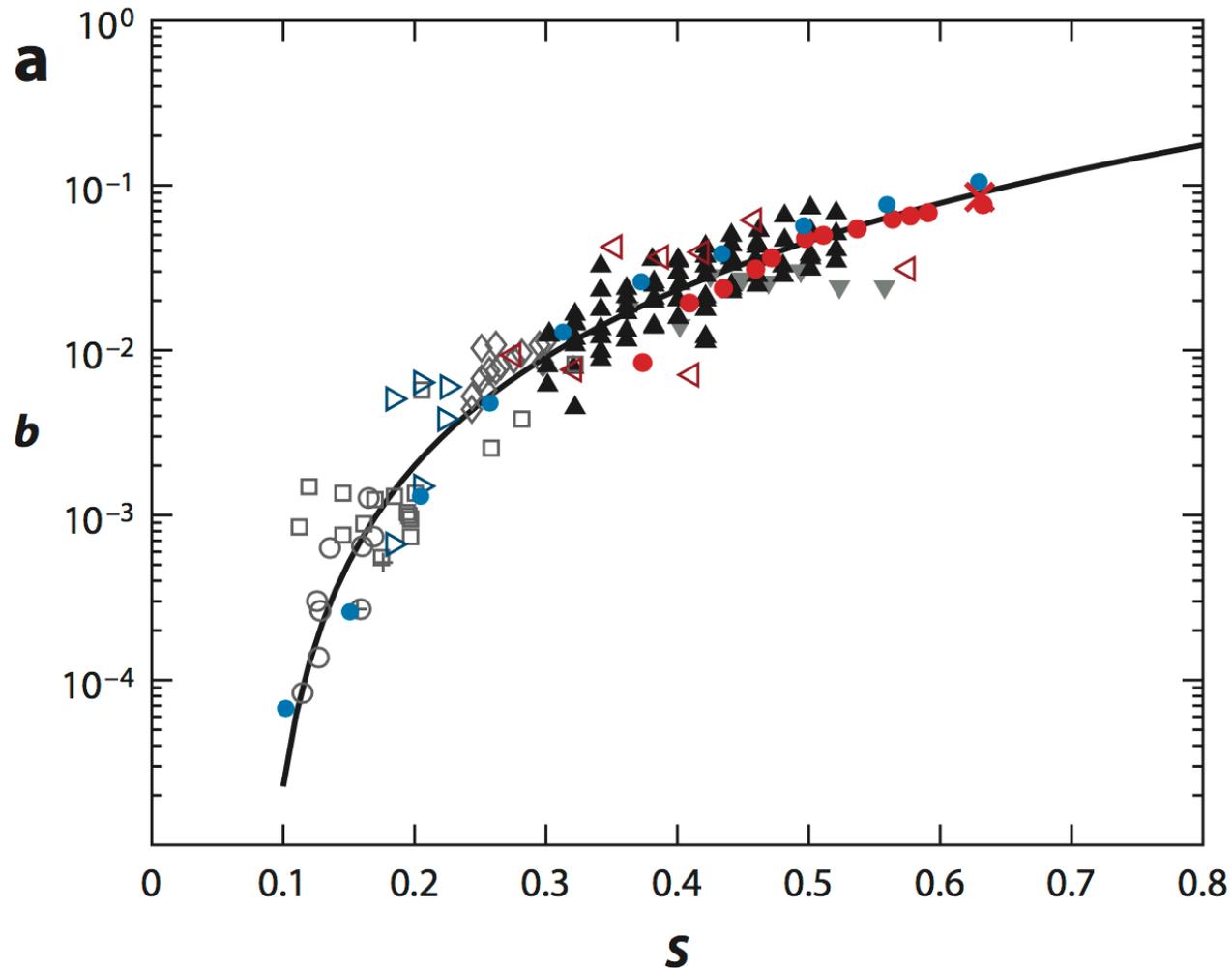


$t = 40$



Direct modeling of breaking statistics using a  
multi-layer model  
(Wu, Popinet and Deike, in prep)

# Energy dissipation due to breaking from DNS are in good agreement with laboratory measurements



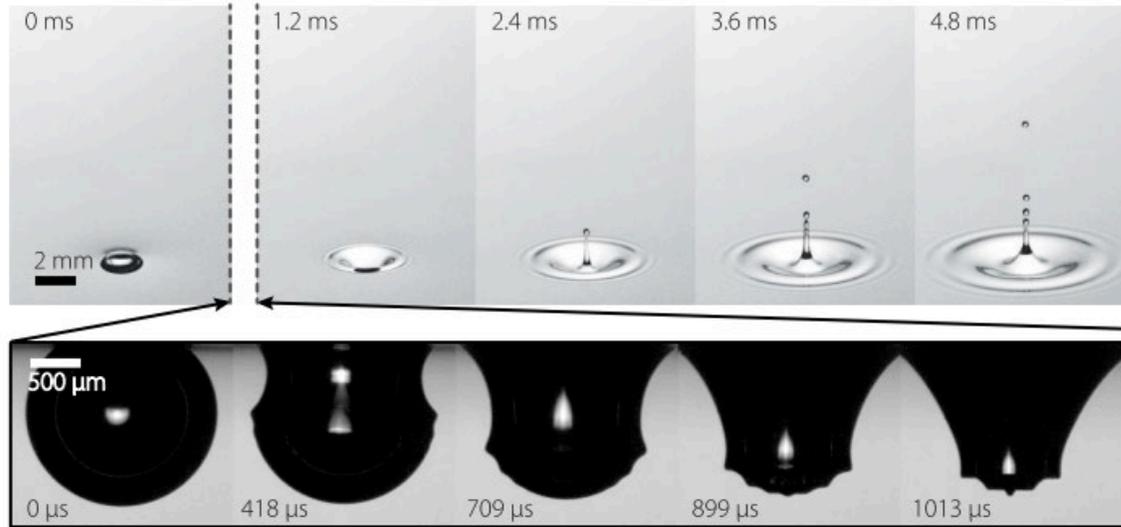
*Adapted from Drazen et al 2008, Romero et al 2012*

- $b = 0.4(S - 0.08)^{5/2}$
- ◇ Melville (1994)
- ▽ Drazen et al. (2008), THL
- ▲ Drazen et al. (2008), SIO
- Grare et al. (2013)
- Banner & Peirson (2007)
- + Banner & Peirson (2007), wide basin
- DNS, Deike et al. (2016)
- ◁ LES, Derakhti & Kirby (2016)
- ▷ DNS 2D, De Vita et al. (2018)
- DNS 2D, Deike et al. (2015)
- ✕ DNS, Mostert et al. (2021)

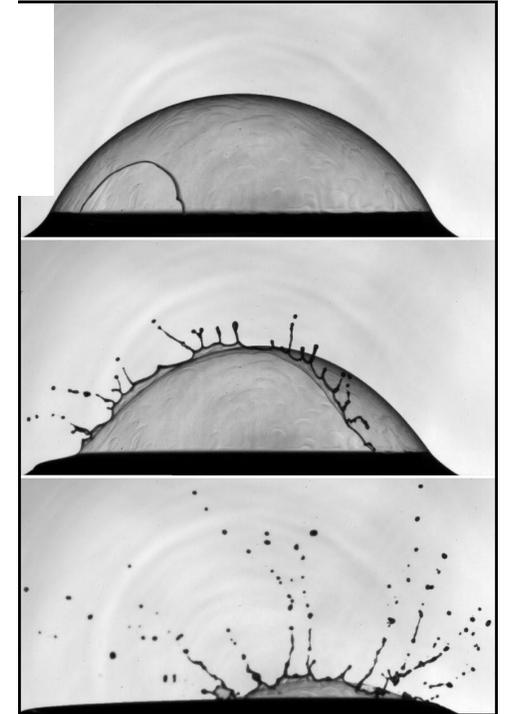
Validates the use DNS to study the multi-phase turbulent flow resulting from breaking

# Scales of production of film and jet drops

Jet drops,  
Ghabache et al 2014



Film drops,  
Lhuissier and  
Villermaux 2012



Visco-capillary length:  $l_\mu = \mu^2 / (\rho\gamma)$

Capillary length:  $l_c = \sqrt{\gamma / (\rho g)}$

